

# Petrov type I Condition and Rindler Fluid in Vacuum Einstein-Gauss-Bonnet Gravity

Rong-Gen Cai<sup>1\*</sup>, Qing Yang<sup>1†</sup>, Yun-Long Zhang<sup>1‡</sup>

<sup>1</sup>State Key Laboratory of Theoretical Physics, Institute of Theoretical Physics,  
Chinese Academy of Sciences, Beijing 100190, People's Republic of China

August 28, 2014

## Abstract

Recently the Petrov type I condition is introduced to reduce the degrees of freedom in the extrinsic curvature of a timelike hypersurface to the degrees of freedom in the dual Rindler fluid in Einstein gravity. In this paper we show that the Petrov type I condition holds for the solutions of vacuum Einstein-Gauss-Bonnet gravity up to the second order in the relativistic hydrodynamic expansion. On the other hand, if imposing the Petrov type I condition and Hamiltonian constraint on a finite cutoff hypersurface, the stress tensor of the relativistic Rindler fluid in vacuum Einstein-Gauss-Bonnet gravity can be recovered with correct first order and second order transport coefficients.

## Contents

<b>1</b>	<b>Introduction</b>	<b>2</b>
<b>2</b>	<b>Rindler fluid in Einstein-Gauss-Bonnet gravity</b>	<b>3</b>
2.1	Rindler fluid in relativistic hydrodynamic expansion . . . . .	4
2.2	The solution is Petrov type I . . . . .	6
<b>3</b>	<b>Petrov type I condition on the hypersurface <math>\Sigma_c</math></b>	<b>7</b>
<b>4</b>	<b>From Petrov type I condition to Rindler fluid</b>	<b>10</b>
4.1	Recover the Rindler fluid in vacuum Einstein gravity . . . . .	11
4.2	Recover the Rindler fluid in Einstein-Gauss-Bonnet gravity . . . . .	13

---

\*E-mail: cairg@itp.ac.cn

†E-mail: yangqing@itp.ac.cn

‡E-mail: zhangyl@itp.ac.cn

<b>5</b>	<b>The non-relativistic hydrodynamic expansion</b>	<b>14</b>
5.1	Petrov type I condition in Rindler fluid . . . . .	15
5.2	Recover the Gauss-Bonnet corrections . . . . .	16
<b>6</b>	<b>Conclusion</b>	<b>17</b>
<b>A</b>	<b>Classification of the Weyl tensor</b>	<b>18</b>
<b>B</b>	<b>Decomposition of the Riemann tensor</b>	<b>20</b>

# 1 Introduction

There has been increasing interest on the holographic duality between fluid dynamics and gravity in the past few years, while the suggestion of such a connection can be dated back to the 1970s proposed by Damour [1, 2]. The approach is developed into the membrane paradigm [3], which relates the black hole evolution and diffusion with those in hydrodynamics [4, 5, 6, 7, 8]. In recent years, along with the progress in the anti-de Sitter/Conformal Field Theory (AdS/CFT) correspondence [9, 10, 11, 12], the dual fluid has been generalized to the conformal fluid living on the boundary of AdS spacetime, which can describe the long wavelength and low frequency limit of conformal field theory [13, 14, 15]. In particular, a systematic method to study the duality was proposed in the fluid/gravity correspondence [16], which translates problems in fluid dynamics into problems in general relativity. It was then further expanded to arbitrary dimensions in [17, 18, 19] and to non-relativistic hydrodynamics in [20].

To build up the connection between the fluid/gravity correspondence and membrane paradigm, a timelike hypersurface outside the horizon is introduced to study the universality of the hydrodynamic limit in AdS/CFT correspondence and membrane paradigm [21, 22, 23]. Significantly, the authors in [23] consider the fluid living on the finite cutoff hypersurface from the viewpoint of Wilsonian renormalization, there Dirichlet boundary condition on the hypersurface and the regularity on the horizon are imposed. Then the fluid/gravity correspondence on the cutoff hypersurface can be generalized to either asymptotically flat [24, 25] or de Sitter spacetime [26], and it has been further studied in [27, 28, 29, 30, 31, 32, 33, 34, 35]. More general discussions in fluid/gravity correspondence can also be found in [36, 37, 38, 39, 40], as well as in the frame of AdS/Ricci-flat correspondence [41, 42].

In the fluid/gravity duality, one of the most important developments is the so-called Rindler hydrodynamics [24, 43, 44, 45, 46, 47, 48], where the dual fluid lives on a constant acceleration hypersurface with a flat induced metric. More interestingly, it is found in [49] that in the near-horizon limit, instead of the regularity condition on the horizon, imposing the Petrov type I condition on the hypersurface can reduce the vacuum Einstein equations to the incompressible Navier-Stokes equations in one lower dimensional flat spacetime. It is mathematically much simpler than solving gravitational field equations. Further study based on this framework can be found in [50, 51, 52, 53, 54, 55]. From the point of view of

degrees of freedom, the Petrov type I condition gives  $(p+2)(p-1)/2$  constraints on the extrinsic curvature of a  $p+1$  dimensional timelike hypersurface, or equivalently on the dual Brown-York stress tensor. Then the degrees of freedom of the stress tensor are reduced to be  $p+2$ , which can be interpreted as energy density, pressure and velocity field of dual fluid [49]. Furthermore the momentum constraint turns out to be the equation of motion of the dual fluid, and the Hamiltonian constraint can be interpreted as the equation of state.

Recently, it has been shown in [56, 57] that, the Petrov type I condition can be used to recover the stress tensor of the dual fluid on the hypersurface order by order under appropriate gauge choice. Without solving the perturbative gravitational field equations, the Rindler fluid in vacuum Einstein gravity can be recovered at least up to the second order in the relativistic hydrodynamic expansion [57]. Note that the stress tensor of Rindler fluid in vacuum Einstein-Gauss-Bonnet gravity is found to be modified by the Gauss-Bonnet coefficient  $\alpha$  in [44, 47]. It is then quite interesting to ask whether the Petrov type I condition holds or not in the vacuum Einstein-Gauss-Bonnet gravity and whether it can be used to recover the dual stress tensor. In this paper, we find that the Petrov type I condition for the solution of vacuum Einstein-Gauss-Bonnet equations still holds up to the second order in the relativistic hydrodynamic expansion, and that turn the logic around, imposing the Petrov type I condition and Hamiltonian constraint, the stress tensor of the relativistic Rindler fluid can be recovered with correct first order and second order transport coefficients including the Gauss-Bonnet term corrections. To be specific, in section 2, we firstly review the Rindler fluid in vacuum Einstein-Gauss-Bonnet gravity, and show that the spacetime with perturbation is at least Petrov type I up to the second order in the relativistic hydrodynamic expansion. In section 3, we give a detailed derivation of the Petrov type I condition on a cutoff hypersurface in vacuum Einstein-Gauss-Bonnet gravity. In section 4, we turn the logic around and assume the Hamiltonian constraint and Petrov type I condition on a finite cutoff hypersurface to recover the stress tensor of the dual fluid without using the details of the solution. We further study the Petrov type I condition in non-relativistic hydrodynamic expansion in section 5, and make the conclusion in section 6.

## 2 Rindler fluid in Einstein-Gauss-Bonnet gravity

To study the fluid dual to vacuum Einstein-Gauss-Bonnet gravity, we begin with the Einstein-Hilbert action on a  $(p+2)$  dimensional Lorentz manifold  $\mathcal{M}$ , with the Gauss-Bonnet term  $\mathcal{L}_{GB} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\sigma\lambda}R^{\mu\nu\sigma\lambda}$  and appropriate surface term [58]

$$S = \frac{1}{16\pi G_{p+2}} \int d^{p+2}x \sqrt{-g} (R - 2\Lambda + \alpha \mathcal{L}_{GB}) + S_{\partial\mathcal{M}}. \quad (1)$$

where  $\alpha$  is the Gauss-Bonnet coefficient. Varying this action with respect to the metric  $g_{\mu\nu}$  yields the vacuum Einstein-Gauss-Bonnet field equations,

$$G_{\mu\nu} + 2\alpha H_{\mu\nu} = 0, \quad G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}, \quad \mu, \nu = 0, 1, \dots, p+1, \quad (2)$$

$$H_{\mu\nu} \equiv RR_{\mu\nu} - 2R_{\mu\lambda}R^\lambda_\nu - 2R^{\sigma\lambda}R_{\mu\sigma\nu\lambda} + R_\mu^{\sigma\lambda\rho}R_{\nu\sigma\lambda\rho} - \frac{1}{4}g_{\mu\nu}\mathcal{L}_{GB}. \quad (3)$$

The  $p+2$  dimensional Rindler metric

$$ds_{p+2}^2 = -r d\tau^2 + 2d\tau dr + \delta_{ij}dx^i dx^j, \quad i, j = 1, \dots, p, \quad (4)$$

is an exact solution of the field equations (2). On a timelike hypersurface  $\Sigma_c$  with  $r = r_c$ , the induced metric is intrinsic flat,

$$ds_{p+1}^2 = \gamma_{ab}dx^a dx^b = -r_c d\tau^2 + dx_i dx^i, \quad a, b = 0, 1, \dots, p. \quad (5)$$

And after setting  $16\pi G_{p+2} = 1$ , the Brown-York stress tensor of Einstein-Gauss-Bonnet gravity on the cutoff surface  $\Sigma_c$  can be written as [59, 27],

$$T_{ab}^{(GB)} = -2(K_{ab} - K\gamma_{ab}) - 4\alpha(3J_{ab} - J\gamma_{ab}), \quad J \equiv \gamma^{ab}J_{ab}, \quad (6)$$

$$J_{ab} \equiv \frac{1}{3}(2KK_{ac}K^c_b + K_{cd}K^{cd}K_{ab} - 2K_{ac}K^{cd}K_{db} - K^2K_{ab}). \quad (7)$$

Here  $K_{ab}$  is the extrinsic curvature of the hypersurface  $\Sigma_c$ .

## 2.1 Rindler fluid in relativistic hydrodynamic expansion

In order to study the dual fluid on the hypersurface  $\Sigma_c$ , one introduces the  $(p+1)$  independent parameters  $u^a = \gamma_v(1, v^i)$  and  $\mathbb{p}$ , which are slowly varying functions of  $x^a = (\tau, x^i)$ . Here  $\gamma_v$  is fixed through  $\gamma_{ab}u^a u^b = -1$ . Keep the induced metric on a timelike hypersurface  $\Sigma_c$  flat and regularity on the future horizon, the solution of vacuum Einstein-Gauss-Bonnet field equation (2) up to the second order in the derivative expansion is given by [45, 46],

$$ds_{p+2}^2 = g_{\mu\nu}dx^\mu dx^\nu = -2\mathbb{p}u_a dx^a dr + g_{ab}dx^a dx^b, \quad (8)$$

$$g_{ab} = g_{ab}^{(0)} + g_{ab}^{(1)} + g_{ab}^{(2)} + O(\partial^3). \quad (9)$$

The leading order term of  $g_{ab}$  in the derivative expansion is

$$g_{ab}^{(0)} = [1 - \mathbb{p}^2(r - r_c)]u_a u_b + h_{ab}, \quad (10)$$

where the projection tensor  $h_{ab} \equiv \gamma_{ab} + u_a u_b$ . We can read out the horizon position through  $r_h = r_c - 1/\mathbb{p}^2$  with  $g_{ab}^{(0)}$  in the case of equilibrium state. The first order term of  $g_{ab}$  in the derivative expansion is

$$g_{ab}^{(1)} = 2\mathbb{p}(r - r_c) [(D \ln \mathbb{p})u_a u_b + 2a_{(a}u_{b)}], \quad (11)$$

where  $D \equiv u^c \partial_c$  and the acceleration  $a^a \equiv u^b \partial_b u^a$ . At the second order in the derivative expansion, the Gauss-Bonnet corrections appear in the metric [47],

$$u^c u^d g_{cd}^{(2)} = +2(r-r_c) \mathcal{K}_{cd} \mathcal{K}^{cd} + \frac{1}{2} \mathbb{P}^2 (r-r_c)^2 (\mathcal{K}_{cd} \mathcal{K}^{cd} + 2a_c a^c) + \frac{1}{2} \mathbb{P}^4 (r-r_c)^3 \Omega_{cd} \Omega^{cd} \\ + 2\alpha \mathbb{P}^2 (r-r_c) \left( \mathcal{K}_{cd} \mathcal{K}^{cd} - \frac{6}{p} \Omega_{cd} \Omega^{cd} \right) + 3\alpha \mathbb{P}^4 (r-r_c)^2 \frac{p-2}{p} \Omega_{cd} \Omega^{cd}, \quad (12)$$

$$h_a^c u^d g_{cd}^{(2)} = -2(r-r_c) h_a^c \partial_d \mathcal{K}_c^d + \mathbb{P}^2 (r-r_c)^2 [h_a^b \partial_c \mathcal{K}_b^c - (\mathcal{K}_{ad} + \Omega_{ad}) a^d], \quad (13)$$

$$h_a^c h_b^d g_{cd}^{(2)} = +2(r-r_c) (-\mathcal{K}_a^c \mathcal{K}_{cb} + 2\mathcal{K}_{c(a} \Omega_{b)}^c - 2h_a^c h_b^d D \mathcal{K}_{cd}) - \mathbb{P}^2 (r-r_c)^2 \Omega_{ac} \Omega_b^c \\ + 12\alpha \mathbb{P}^2 (r-r_c) \left[ \Omega_{ac} \Omega_b^c + \frac{1}{p} (\Omega_{cd} \Omega^{cd}) h_{ab} \right]. \quad (14)$$

Here the fluid shear and vorticity are defined as

$$\mathcal{K}_{ab} \equiv h_a^c h_b^d \partial_{(c} u_{d)}, \quad \Omega_{ab} \equiv h_a^c h_b^d \partial_{[c} u_{d]}. \quad (15)$$

The components of inverse metric up to the second order in the derivative expansion are

$$g^{rr} = \mathbb{P}^{-2} \left[ 1 + \mathbb{P}^2 (r-r_c) - \left( g_{cd}^{(1)} + g_{cd}^{(2)} - h^{ab} g_{ac}^{(1)} g_{bd}^{(1)} \right) u^c u^d \right], \\ g^{ra} = \mathbb{P}^{-1} \left( u^a + h^{ab} g_{bc}^{(1)} u^c + h^{ab} g_{bc}^{(2)} u^c \right), \\ g^{ab} = h^{ab} - h^{ac} h^{bd} g_{cd}^{(2)}. \quad (16)$$

one also needs to consider the following constraint equations

$$\partial_a u^a = 2\mathbb{P}^{-1} \mathcal{K}_{ab} \mathcal{K}^{ab} + O(\partial^3), \\ a_a + D_a^\perp \ln \mathbb{P} = 2\mathbb{P}^{-1} h_a^c \partial_b \mathcal{K}_c^b + O(\partial^3), \quad (17)$$

with  $D_a^\perp \equiv h_a^c \partial_c$ , so that the metric (8) solves the vacuum Einstein-Gauss-Bonnet field equations (2) up to the second order in the derivative expansion.

With the metric (8) and appropriate gauge choice, the dual stress tensor  $T_{ab}^{(GB)}$  in the vacuum Einstein-Gauss-Bonnet gravity on the finite cutoff surface  $\Sigma_c$  in (6) has been obtained in [46],

$$T_{ab}^{(GB)} = +\mathbb{P} h_{ab} - 2\mathcal{K}_{ab} - 2\mathbb{P}^{-1} (\mathcal{K}_{ab} \mathcal{K}^{ab}) u_a u_b \\ + \mathbb{P}^{-1} \left[ -2(1 + 2\alpha \mathbb{P}^2) \mathcal{K}_{ac} \mathcal{K}_b^c - 4\mathcal{K}_{c(a} \Omega_{b)}^c - 4(1 + 3\alpha \mathbb{P}^2) \Omega_{ac} \Omega_b^c \right. \\ \left. - 4h_a^c h_b^d \partial_c \partial_d \ln \mathbb{P} - 4\mathcal{K}_{ab} D \ln \mathbb{P} + 4(D_a^\perp \ln \mathbb{P})(D_b^\perp \ln \mathbb{P}) \right]. \quad (18)$$

On the other hand, the general stress tensor  $T_{ab}^{(R)}$  for  $(p+1)$ -dimensional relativistic fluid with vanishing equilibrium energy density is constructed in [45] as

$$T_{ab}^{(R)} = +\mathbb{P} h_{ab} - 2\eta \mathcal{K}_{ab} + \zeta' (D \ln \mathbb{P}) u_a u_b \\ + \mathbb{P}^{-1} \left[ d_1 \mathcal{K}_{cd} \mathcal{K}^{cd} + d_2 \Omega_{cd} \Omega^{cd} + d_3 (D \ln \mathbb{P})^2 + d_4 D D \ln \mathbb{P} + d_5 (D^\perp \ln \mathbb{P})^2 \right] u_a u_b \\ + \mathbb{P}^{-1} \left[ c_1 \mathcal{K}_{ac} \mathcal{K}_b^c + c_2 \mathcal{K}_{c(a} \Omega_{b)}^c + c_3 \Omega_{ac} \Omega_b^c + c_4 h_a^c h_b^d \partial_c \partial_d \ln \mathbb{P} + c_5 \mathcal{K}_{ab} D \ln \mathbb{P} \right. \\ \left. + c_6 D_a^\perp \ln \mathbb{P} D_b^\perp \ln \mathbb{P} \right]. \quad (19)$$

Compare  $T_{ab}^{(GB)}$  in (18) with  $T_{ab}^{(R)}$ , one can read out the holographic transport coefficients of Rindler fluid in vacuum Einstein-Gauss-Bonnet gravity as

$$\begin{aligned} \zeta' &= 0, & \eta &= 1, & d_1 &= -2, & d_2 &= d_3 = d_4 = d_5 = 0, \\ c_1 &= -2(1 + 2\alpha\mathbb{P}^2), & c_3 &= -4(1 + 3\alpha\mathbb{P}^2), & c_2 &= c_4 = c_5 = -c_6 = -4. \end{aligned} \quad (20)$$

It turns out that there are no Gauss-Bonnet corrections to the shear viscosity  $\eta$  and the parameter  $\zeta'$ , the latter measures variations of the energy density. The Gauss-Bonnet corrections appear in the second order transport coefficients  $c_1$  and  $c_3$ .

## 2.2 The solution is Petrov type I

The Petrov type classification of Weyl tensor in higher dimensions is summarized in Appendix A. In this subsection, we will show that the Weyl tensors  $C_{\mu\nu\alpha\beta}$  of the metric  $g_{\mu\nu}$  in (8) is at least Petrov type I.

Choose  $(p+2)$  Newman-Penrose-like vector fields, which include two null vectors  $\ell^2 = k^2 = 0$ , and  $p$  orthonormal space-like vectors  $\mathbf{m}_i$ . The null vectors obey  $\ell_\mu k^\mu = 1$  and all other products with  $\mathbf{m}_i (i = 1, \dots, p)$  vanish. Define

$$\mathbb{P}_{ij}^{(r)} \equiv 2C_{(\ell)i(\ell)j} \equiv 2\ell^\mu \mathbf{m}_i{}^\nu \ell^\alpha \mathbf{m}_j{}^\beta C_{\mu\nu\alpha\beta}. \quad (21)$$

Then the Weyl tensor  $C_{\mu\nu\alpha\beta}$  is at least Petrov type I if there exists a frame  $\{\ell, k, \mathbf{m}_i\}$  such that  $\mathbb{P}_{ij}^{(r)} = 0$ . A special kind of frame has been chosen in [57]. If we denote  $\mathbf{n}$  as the spacelike unit normal vector of a constant  $r$  hypersurface,  $\mathbf{u}$  is the normalized  $(p+2)$  velocity along with the hypersurface, the two null vector fields can be chosen as their combinations

$$\sqrt{2}\ell = -\mathbf{n} + \mathbf{u}, \quad \sqrt{2}k = -\mathbf{n} - \mathbf{u}. \quad (22)$$

For the remaining orthonormal spatial vectors  $\mathbf{m}_i$ , there exists still a freedom to choose. Consider the fact that  $m_i^a m_b^i = h_b^a = \delta_b^a + u^a u_b$ , and

$$\begin{aligned} m_i^a &= \delta_i^a + r_c^{-1/2} u_i \delta_\tau^a + (1 + r_c^{1/2} \gamma_v)^{-1} u_i u^j \delta_j^a, \\ m_a^i &= \delta_a^i - r_c^{+1/2} u^i \delta_a^\tau + (1 + r_c^{1/2} \gamma_v)^{-1} u^i u_j \delta_a^j, \end{aligned} \quad (23)$$

the components of the frame have been chosen as follows [57],

$$\begin{aligned} \sqrt{2}\ell^\mu &= -\mathbf{n}^r \delta_r^\mu - (\mathbf{n}^a - \mathbf{u}^a) \delta_a^\mu = (g^{rr})^{1/2} \delta_r^\mu, \\ \sqrt{2}k^\mu &= -\mathbf{n}^r \delta_r^\mu - (\mathbf{n}^a + \mathbf{u}^a) \delta_a^\mu = -(g^{rr})^{1/2} (\delta_r^\mu + 2g^{ra} \delta_a^\mu), \\ \mathbf{m}_i{}^\mu &= \mathbf{m}_i^a \delta_a^\mu = \left( m_i^a - \frac{1}{2} m_i^b g_{bc}^{(2)} h^{ca} \right) \delta_a^\mu. \end{aligned} \quad (24)$$

And the components with subscript index are

$$\begin{aligned}\sqrt{2}\ell_\mu &= -(\mathbf{n}_r - \mathbf{u}_r)\delta_\mu^r + \mathbf{u}_a\delta_\mu^a = (g^{rr})^{-1/2}\mathbb{P}u_a\delta_\mu^a, \\ \sqrt{2}\mathbf{k}_\mu &= -(\mathbf{n}_r + \mathbf{u}_r)\delta_\mu^r - \mathbf{u}_a\delta_\mu^a = -2(g^{rr})^{-1/2}\delta_\mu^r - (g^{rr})^{-1/2}\mathbb{P}u_a\delta_\mu^a, \\ \mathbf{m}_\mu^i &= \left[ m_a^i + u_a u^b (g_{bc}^{(1)} + g_{bc}^{(2)}) h^{cd} m_d^i - \frac{1}{2} h_a^b g_{bc}^{(2)} h^{cd} m_d^i \right] \delta_\mu^a.\end{aligned}\quad (25)$$

Up to order  $\partial^2$ , one can check that  $g_{\mu\nu}\mathbf{m}_i^\mu\mathbf{m}_j^\nu = \delta_{ij}$  is satisfied, and metric (8) as well as its inverse (16) can be decomposed as

$$g_{\mu\nu} = 2\ell_{(\mu}\mathbf{k}_{\nu)} + \delta_{ij}\mathbf{m}_\mu^i\mathbf{m}_\nu^j, \quad g^{\mu\nu} = 2\ell^{(\mu}\mathbf{k}^{\nu)} + \delta^{ij}\mathbf{m}_i^\mu\mathbf{m}_j^\nu. \quad (26)$$

To check the Petrov type I condition  $\mathbb{P}_{ij}^{(r)} = 0$  of the Weyl tensor, we introduce another covariant formula  $\mathbb{P}_{ab}^{(r)}$ , which is defined as

$$\mathbb{P}_{ab}^{(r)} \equiv 2h_a^c h_b^d C_{(\ell)c(\ell)d} = \mathbf{n}^r h_a^c \mathbf{n}^r h_b^d C_{rcrd}, \quad \mathbb{P}_{ij}^{(r)} = \mathbf{m}_i^a \mathbf{m}_j^b \mathbb{P}_{ab}^{(r)}. \quad (27)$$

Then after a straightforward calculation of the Weyl tensors with metric (8), we find

$$\mathbb{P}_{ab}^{(r)} = -g^{rr} \left( \frac{1}{2} h_a^c h_b^d \partial_r^2 g_{cd}^{(2)} + \mathbb{P}^2 \Omega_{ac} \Omega_b^c \right) + O(\partial^3). \quad (28)$$

Considering  $g_{cd}^{(2)}$  with Gauss-Bonnet corrections in (14), we can conclude that  $\mathbb{P}_{ab}^{(r)} = O(\partial^3)$  at arbitrary  $r$ , which also indicates  $\mathbb{P}_{ij}^{(r)} = O(\partial^3)$  at every spacetime point in (8). As a result, we have shown that the Weyl tensor or the spacetime with metric (8) is at least Petrov type I up to  $\partial^2$ , even when the Gauss-Bonnet term is included.

### 3 Petrov type I condition on the hypersurface $\Sigma_c$

The Petrov type I condition is introduced to reduce the degrees of freedom in the extrinsic curvature of the hypersurface  $\Sigma_c$  to the degrees of freedom in the dual fluid on  $\Sigma_c$  in [49]. On this hypersurface, the covariant Petrov type I condition is defined as [57],

$$\mathbb{P}_{ab} \equiv \mathbb{P}_{ab}^{(r_c)} = 2h_a^c h_b^d C_{(\ell)c(\ell)d}|_{\Sigma_c} = 0. \quad (29)$$

With (22) and consider the fact that

$$2C_{(\ell)c(\ell)d} = C_{(\mathbf{u})c(\mathbf{u})d} - C_{(\mathbf{u})c(\mathbf{n})d} - C_{(\mathbf{u})d(\mathbf{n})c} + C_{(\mathbf{n})c(\mathbf{n})d}, \quad (30)$$

we need to rewrite the Weyl tensor in terms of the extrinsic curvature  $K_{ab}$ , through using the Gauss-Codazzi equations on the intrinsic flat hypersurface  $\Sigma_c$ . Thus, we firstly define the following notations

$$\begin{aligned}M_{abcd} &\equiv \gamma_a^\alpha \gamma_b^\beta \gamma_c^\gamma \gamma_d^\delta R_{\alpha\beta\gamma\delta} = K_{ad}K_{bc} - K_{ac}K_{bd}, \\ N_{abc} &\equiv \gamma_a^\alpha \gamma_b^\beta \gamma_c^\gamma n^\delta R_{\alpha\beta\gamma\delta} = \partial_a K_{bc} - \partial_b K_{ac}, \\ Y_{ab} &\equiv \gamma_a^\alpha n^\beta \gamma_b^\gamma n^\delta R_{\alpha\beta\gamma\delta} = K K_{ab} - K_{ac}K_b^c + \gamma_a^\alpha \gamma_b^\gamma R_{\alpha\gamma},\end{aligned}\quad (31)$$

with  $\gamma_a^\alpha = \delta_a^\alpha - n_a n^\alpha = \delta_a^\alpha$ , as well as their contractions,

$$\begin{aligned} M_{ac} &\equiv \gamma^{bd} M_{abcd} = K_{ab} K^b{}_c - K K_{ac}, & N_b &\equiv \gamma^{ac} N_{abc} = \partial_a (K^a{}_b - K \gamma^a_b), \\ M &\equiv \gamma^{ac} M_{ac} = K_{ab} K^{ab} - K^2, & Y &\equiv \gamma^{ac} Y_{ac} = -M + \gamma^{\alpha\beta} R_{\alpha\beta}. \end{aligned} \quad (32)$$

Then using the equations of motion (2) which lead to

$$R_{\mu\nu} = -\frac{2}{p} \alpha H g_{\mu\nu} - 2\alpha H_{\mu\nu}, \quad R = \frac{4}{p} \alpha H, \quad H \equiv H_{\mu\nu} g^{\mu\nu}, \quad (33)$$

we can obtain the projections of the Weyl tensor on the hypersurface  $\Sigma_c$ ,

$$\begin{aligned} \gamma_a^\alpha \gamma_b^\beta \gamma_c^\gamma \gamma_d^\delta C_{\alpha\beta\gamma\delta} &= M_{abcd} - \frac{8\alpha H}{p(p+1)} \gamma_{a[c} \gamma_{d]b} + \alpha \frac{4}{p} \gamma_a^\alpha \gamma_b^\beta \gamma_c^\gamma \gamma_d^\delta (g_{\alpha[\gamma} H_{\delta]\beta} - g_{\beta[\gamma} H_{\delta]\alpha}), \\ \gamma_a^\alpha \gamma_b^\beta \gamma_c^\gamma n^\delta C_{\alpha\beta\gamma\delta} &= N_{abc} + \alpha \frac{4}{p} \gamma_a^\alpha \gamma_b^\beta \gamma_c^\gamma n^\delta (g_{\alpha[\gamma} H_{\delta]\beta} - g_{\beta[\gamma} H_{\delta]\alpha}), \\ \gamma_a^\alpha n^\beta \gamma_c^\gamma n^\delta C_{\alpha\beta\gamma\delta} &= Y_{ac} - \frac{4\alpha H}{p(p+1)} \gamma_{ac} + \alpha \frac{4}{p} \gamma_a^\alpha n^\beta \gamma_c^\gamma n^\delta (g_{\alpha[\gamma} H_{\delta]\beta} - g_{\beta[\gamma} H_{\delta]\alpha}). \end{aligned} \quad (34)$$

This is similar to the derivation in [52] for the case of Einstein gravity with matter. Then put (34) into (29) and consider (30), we obtain  $\mathbb{P}_{ab} = \mathbb{P}_{ab}^{(\alpha)} + \delta\mathbb{P}_{ab}^{(H)}$ , where

$$\mathbb{P}_{ab}^{(\alpha)} \equiv M_{(u)a(u)b}^\perp + 2N_{(u)(ab)}^\perp - M_{ab}^\perp, \quad (35)$$

$$\delta\mathbb{P}_{ab}^{(H)} \equiv -2\alpha H_{ab}^\perp + 2\alpha p^{-1} [H_{(n)(n)} - 2H_{(n)(u)} + H_{(u)(u)} + H] h_{ab}. \quad (36)$$

For convenience, we here have defined

$$M_{(u)a(u)b}^\perp = h_a^m h_b^n M_{cmdn} u^c u^d, \quad N_{(u)(ab)}^\perp = h_a^m h_b^n N_{cmn} u^c, \quad M_{ab}^\perp = h_a^m h_b^n M_{mn}, \quad (37)$$

as well as

$$\begin{aligned} H_{ab}^\perp &\equiv H_{\mu\nu} \gamma_a^\mu \gamma_b^\nu h_a^c h_b^d, & H_{(n)(n)} &\equiv H_{\mu\nu} n^\mu n^\nu, \\ H_{(u)(u)} &\equiv H_{\mu\nu} \gamma_a^\mu \gamma_b^\nu u^a u^b, & H_{(n)(u)} &\equiv H_{\mu\nu} n^\mu \gamma_b^\nu u^b. \end{aligned} \quad (38)$$

On the other hand, the Hamiltonian constraint for the vacuum Einstein-Gauss-Bonnet field equations (2) is

$$\mathbb{H} \equiv -2(G_{\mu\nu} + 2\alpha H_{\mu\nu}) n^\mu n^\nu = 0. \quad (39)$$

With the decomposition of the Riemann tensor in Appendix B, we obtain  $\mathbb{H} = \mathbb{H}^{(\alpha)} + \delta\mathbb{H}^{(H)}$ , where [60]

$$\mathbb{H}^{(\alpha)} \equiv M, \quad \delta\mathbb{H}^{(H)} \equiv \alpha (M^2 - 4M_{ab} M^{ab} + M_{abcd} M^{abcd}). \quad (40)$$

While the momentum constraint for the equations of motion (2) turns out to be

$$\partial^a T_{ab}^{(GB)} \equiv -2(E_{\mu\nu} + 2\alpha H_{\mu\nu}) n^\mu \gamma_b^\nu = 0, \quad (41)$$



where  $T_{ab}^{(GB)}$  is the one given in (6).

Notice that  $\mathbb{P}_{ab}^{(\alpha)}$  in (35) has become the hypersurface function of extrinsic curvature  $K_{ab}$ , but it is not true for  $\delta\mathbb{P}_{ab}^{(H)}$  in (36). For example, we can see from [60] that the term

$$Y_{ab} = -M_{ab} + \gamma_a^\mu \gamma_b^\nu R_{\mu\nu} = -\mathcal{L}_n K_{ab} + K_{ac} K_b^c \quad (42)$$

appears in  $2\alpha H_{ab}^\perp$ ,  $Y_{ab}$  can not be obtained only from the extrinsic curvature  $K_{ab}$  and other intrinsic quantities, because additional information of the bulk gravity such as  $R_{\mu\nu}$ , or the analytic continuation of  $K_{ab}$  out of the hypersurface along  $n$  is needed. Thus the purpose of Petrov type I condition that gives constraints to the extrinsic curvature can not be realized in this scene. However, if we consider only the small Gauss-Bonnet parameter  $\alpha$  limit, and take the Petrov type I condition up to the first order in the  $\alpha$  expansion, the above difficulty can be relieved.

To see this, we firstly define all the quantities with bars have the same formulas as those without bars when  $\alpha = 0$ . Then put (33) into (42) and (3), we obtain  $\bar{Y}_{ab} = -\bar{M}_{ab}$ , as well as

$$H_{\mu\nu} = \bar{H}_{\mu\nu} + O(\alpha), \quad \bar{H}_{\mu\nu} \equiv \bar{R}_\mu^{\sigma\lambda\rho} \bar{R}_{\nu\sigma\lambda\rho} - \frac{1}{4} (\bar{R}^{\kappa\sigma\lambda\rho} \bar{R}_{\kappa\sigma\lambda\rho}) \bar{g}_{\mu\nu}. \quad (43)$$

With the calculations in Appendix B, the equation (36) becomes  $\delta\mathbb{P}_{ab}^{(H)} = \delta\bar{\mathbb{P}}_{ab}^{(H)} + O(\alpha^2)$ , where  $\delta\bar{\mathbb{P}}_{ab}^{(H)}$  is the first order in the small  $\alpha$  expansion that

$$\delta\bar{\mathbb{P}}_{ab}^{(H)} \equiv -2\alpha \bar{H}_{ab}^\perp + 2\alpha p^{-1} h_{ab} [\bar{H}_{(n)(n)} - 2\bar{H}_{(n)(u)} + \bar{H}_{(u)(u)} + \bar{H}] \quad (44)$$

$$\begin{aligned} &= -2\alpha h_a^m h_b^n (\bar{M}_m^{cde} \bar{M}_{ncde} + 2\bar{N}_m^{cd} \bar{N}_{ncd} + \bar{N}^{cd}_m \bar{N}_{cdn} + 2\bar{M}_m^d \bar{M}_{nd}) \\ &\quad + \alpha p^{-1} h_{ab} \left[ 2(\bar{M}_{(u)}^{cde} \bar{M}_{(u)cde} + 2\bar{N}_{(u)}^{cd} \bar{N}_{(u)cd} + \bar{N}^{cd}_{(u)} \bar{N}_{cd(u)} + 2\bar{M}_{(u)}^d \bar{M}_{(u)d}) \right. \\ &\quad \left. + 4(\bar{M}_{(u)cde} \bar{N}^{dec} - 2\bar{M}^{cd} \bar{N}_{(u)cd}) + (\bar{M}^{cdef} \bar{M}_{cdef} + 6\bar{N}^{cde} \bar{N}_{cde} + 8\bar{M}^{cd} \bar{M}_{cd}) \right]. \quad (45) \end{aligned}$$

Now we can say that  $\delta\bar{\mathbb{P}}_{ab}^{(H)}$  is a function of  $K_{ab}$ ,  $\gamma_{ab}$  as well as  $u_a$ . On the other hand, notice that the extrinsic curvature  $K_{ab}$  can be decomposed as

$$K_{ab} = \bar{K}_{ab} + \delta K_{ab}^{(\alpha)} + O(\alpha^2), \quad (46)$$

where  $\bar{K}_{ab}$  is the contribution from vacuum Einstein gravity, and  $\delta K_{ab}^{(\alpha)}$  includes the terms from the Gauss-Bonnet term at first order in small  $\alpha$  expansion. Then from (35) we have  $\mathbb{P}_{ab}^{(\alpha)} = \bar{\mathbb{P}}_{ab} + \delta\mathbb{P}_{ab}^{(\alpha)} + O(\alpha^2)$ , where

$$\bar{\mathbb{P}}_{ab} \equiv \bar{M}_{(u)a(u)b}^\perp + 2\bar{N}_{(u)(ab)}^\perp - \bar{M}_{ab}^\perp, \quad (47)$$

$$\delta\mathbb{P}_{ab}^{(\alpha)} = \delta M_{(u)a(u)b}^{\perp(\alpha)} + 2\delta N_{(u)(ab)}^{\perp(\alpha)} - \delta M_{ab}^{\perp(\alpha)}. \quad (48)$$

Finally, the covariant Petrov type I condition (29) up to the first order in small  $\alpha$  becomes

$$\mathbb{P}_{ab} \equiv \bar{\mathbb{P}}_{ab} + \delta\mathbb{P}_{ab}^{(\alpha)} + \delta\bar{\mathbb{P}}_{ab}^{(H)} = 0. \quad (49)$$

Similarly, the Hamiltonian constraint (39) up to the first order in small  $\alpha$  becomes,

$$\mathbb{H} = \bar{\mathbb{H}} + \delta\mathbb{H}^{(\alpha)} + \delta\bar{\mathbb{H}}^{(H)} = 0, \quad (50)$$

where

$$\bar{\mathbb{H}} \equiv \bar{M}, \quad \delta\mathbb{H}^{(\alpha)} \equiv \delta M^{(\alpha)}, \quad (51)$$

$$\delta\bar{\mathbb{H}}^{(H)} \equiv \alpha (\bar{M}^2 - 4\bar{M}_{ab}\bar{M}^{ab} + \bar{M}_{abcd}\bar{M}^{abcd}). \quad (52)$$

With the expansion of  $K_{ab}$  in (46), the Brown-York stress tensor (6) can also be expanded as

$$T_{ab}^{(GB)} \equiv \bar{T}_{ab} + \delta T_{ab} + O(\alpha^2), \quad (53)$$

$$\bar{T}_{ab} \equiv -2(\bar{K}_{ab} - \bar{K}\gamma_{ab}), \quad \delta T_{ab} = \delta T_{ab}^{(\alpha)} + \delta\bar{T}_{ab}^{(J)}, \quad (54)$$

where  $\bar{T}_{ab}$  is just the Brown-York stress tensor of Einstein gravity, and  $\delta T_{ab}$  comes from the Gauss-Bonnet term at the first order in small  $\alpha$ ,

$$\delta T_{ab}^{(\alpha)} \equiv -2(\delta K_{ab}^{(\alpha)} - \delta K^{(\alpha)}\gamma_{ab}), \quad \delta\bar{T}_{ab}^{(J)} \equiv -4\alpha(3\bar{J}_{ab} - \bar{J}\gamma_{ab}). \quad (55)$$

In the following section, with the Petrov type I condition (49) and Hamiltonian constraint (50), as well as the stress tensor (53), we will directly recover the stress tensor (18) of Rindler fluid in vacuum Einstein-Gauss-Bonnet gravity.

Notice that in the Einstein gravity,  $\bar{K}_{ab}$  can be expressed in terms of its Brown-York stress tensor through  $\bar{T}_{ab} = 2(\bar{K}\gamma_{ab} - \bar{K}_{ab})$ . But if we consider the Gauss-Bonnet corrections in (6), as the cube terms of  $K_{ab}$  appear in  $J_{ab}$ , one cannot obtain the extrinsic curvature  $K_{ab}$  in terms of the stress tensor  $T_{ab}^{(GB)}$  in (53) at finite  $\alpha$ . But, up to the first order in small  $\alpha$  and from (53), we can have

$$2\bar{K}_{ab} = -\bar{T}_{ab} + p^{-1}\bar{T}\gamma_{ab}, \quad (56)$$

$$2\delta K_{ab}^{(\alpha)} = -\delta T_{ab} + p^{-1}\delta T\gamma_{ab} - 4\alpha(3\bar{J}_{ab} - 2p^{-1}\bar{J}\gamma_{ab}), \quad (57)$$

such that the Petrov type I condition on the hypersurface can also be expressed in terms of the Brown-York stress tensor in Einstein-Gauss-Bonnet gravity  $T_{ab}^{(GB)} = \bar{T}_{ab} + \delta T_{ab}$ . Although it is not necessary in our next section 4.2, the formulas in terms of the stress tensor would be much more in accord with the original purpose of the Petrov type I condition [49]. This also gives us the other motivation to take the small  $\alpha$  limit, and we will use this strategy when study the Petrov type I condition in the non-relativistic hydrodynamic expansion in section 5.

## 4 From Petrov type I condition to Rindler fluid

In this section, we will show how to recover the stress tensor dual to the bulk metric in (8) by use of the Petrov type I condition without the details of the solution (8). We firstly set  $\alpha = 0$  to obtain the Rindler fluid in vacuum Einstein gravity from Petrov type I condition and Hamiltonian constraint. Then regarding  $\alpha$  as a small parameter, the Gauss-Bonnet corrections to the stress tensor up to first order in small  $\alpha$  can also be obtained naturally.

## 4.1 Recover the Rindler fluid in vacuum Einstein gravity

Firstly, setting  $\alpha = 0$  in (49), we have the Petrov type I condition on the finite cutoff hypersurface  $\Sigma_c$  in the vacuum Einstein gravity,

$$\bar{\mathbb{P}}_{ab} \equiv \bar{M}_{(u)a(u)b}^\perp + 2\bar{N}_{(u)(ab)}^\perp - \bar{M}_{ab}^\perp = 0, \quad (58)$$

where similar to (37), we have defined

$$\begin{aligned} \bar{M}_{(u)a(u)b}^\perp &= h_a^m h_b^n (\bar{K}_{cm} \bar{K}_{dn} - \bar{K}_{cd} \bar{K}_{mn}) u^c u^d, \\ \bar{N}_{(u)(ab)}^\perp &= h_{(a}^m h_{b)}^n (u^c \partial_c \bar{K}_{mn} - u^c \partial_m \bar{K}_{nc}), \\ \bar{M}_{ab}^\perp &= -h_a^m h_b^n (\bar{K} \bar{K}_{mn} - \bar{K}_{mc} \bar{K}_n^c). \end{aligned} \quad (59)$$

On the other hand, from (56), we have

$$2\bar{K}_{ab} = -\bar{T}_{ab} + p^{-1} \bar{T} \gamma_{ab}, \quad 2\bar{K} = p^{-1} \bar{T}. \quad (60)$$

Then we can reach the covariant Petrov type I condition that [57]

$$\begin{aligned} 4\bar{\mathbb{P}}_{ab} &= h_a^m h_b^n [\bar{T}_{mc} \bar{T}_{nd} - \bar{T}_{mn} \bar{T}_{cd}] u^c u^d - \bar{T}_{mc} \bar{T}_n^c - 4u^c \partial_c \bar{T}_{mn} + 4u^c \partial_{(m} \bar{T}_{n)c}] \\ &+ p^{-2} [\bar{T}(\bar{T} + p \bar{T}_{cd} u^c u^d) + 4p u^c \partial_c \bar{T}] h_{ab} = 0. \end{aligned} \quad (61)$$

Now we decompose the arbitrary stress tensor  $\bar{T}_{ab}$  associated with a  $(p+1)$ -velocity  $u_a$  as

$$\bar{T}_{ab} = \mathbb{e} u_a u_b + 2\mathbb{j}_{(a} u_{b)} + \Pi_{ab}, \quad \bar{T} = -\mathbb{e} + \Pi. \quad (62)$$

where we have defined

$$\mathbb{e} \equiv \bar{T}_{ab} u^a u^b, \quad \mathbb{j}_a \equiv -h_a^c \bar{T}_{cd} u^d, \quad \Pi_{ab} \equiv h_a^c h_b^d \bar{T}_{cd}, \quad \Pi \equiv \Pi_{ab} h^{ab}. \quad (63)$$

Substituting (62) into (61) we have

$$\begin{aligned} 4\bar{\mathbb{P}}_{ab} &\equiv -\mathbb{e} \Pi_{ab} + 2\mathbb{j}_a \mathbb{j}_b - \Pi_{ac} \Pi_b^c - 8a_{(a} \mathbb{j}_{b)} - 4h_a^c h_b^d D \Pi_{cd} - 4\mathbb{e} \mathcal{K}_{ab} - 4D_{(a}^\perp \mathbb{j}_{b)} - 4\Pi_{(a}^c D_{b)}^\perp u_c \\ &+ p^{-2} [\Pi^2 + (p-2)\mathbb{e} \Pi - (p-1)\mathbb{e}^2 + 4p D(\Pi - \mathbb{e})] h_{ab} = 0. \end{aligned} \quad (64)$$

Similarly, when  $\alpha = 0$ , the Hamiltonian constraint in (50) becomes

$$4\bar{\mathbb{H}} \equiv p \bar{T}_{ab} \bar{T}^{ab} - \bar{T}^2 = 2\mathbb{e} \Pi + (p-1)\mathbb{e}^2 - 2p \mathbb{j}_a \mathbb{j}^a h^{ab} + p \Pi_{ab} \Pi^{ab} - \Pi^2 = 0. \quad (65)$$

Expanding the undetermined stress tensor  $\bar{T}_{ab}$  in (62) in terms of the derivative expansion parameter  $\partial$  as

$$\begin{aligned} \mathbb{e} &= \mathbb{e}^{(0)} + \mathbb{e}^{(1)} + \mathbb{e}^{(2)} + O(\partial^3), \\ \mathbb{j}_a &= \mathbb{j}_a^{(0)} + \mathbb{j}_a^{(1)} + \mathbb{j}_a^{(2)} + O(\partial^3), \\ \Pi_{ab} &= \Pi_{ab}^{(0)} + \Pi_{ab}^{(1)} + \Pi_{ab}^{(2)} + O(\partial^3), \\ \Pi &= \Pi^{(0)} + \Pi^{(1)} + \Pi^{(2)} + O(\partial^3), \end{aligned} \quad (66)$$

and assuming that the zeroth order of the stress tensor has the same form as that in the Rindler fluid (19),

$$\mathfrak{e}^{(0)} = 0, \quad \mathfrak{j}_a^{(0)} = 0, \quad \Pi_{ab}^{(0)} = \mathbb{P}h_{ab}, \quad \Pi^{(0)} = p\mathbb{P}, \quad (67)$$

we can recover the first and second order terms of total stress tensor (18) with  $\alpha = 0$ , by imposing the Hamiltonian constraint (65) and Petrov type I condition (64). As there is an arbitrary for frame choice of the fluid velocity, we define the relativistic fluid velocity  $u^a$  such that  $\mathfrak{j}_a = u^c \bar{T}_{cd} h_a^d \equiv 0$  at arbitrary orders, and choose appropriate isotropy gauge that there is no higher order correction to the term which is proportional to  $h_{ab}$ , that is only  $\mathbb{P}h_{ab}$  appears in the stress tensor [45]. To be specific, we can go as follows.

**i) First order.**

We put (66) and (67) into the Hamiltonian constraint (65) and Petrov type I condition (64), and then expand them in the derivative expansion. Assuming  $\mathfrak{j}_a^{(1)} = 0$ , at the first order, we have

$$\bar{\mathbb{H}}^{(1)} = 0 \Rightarrow \mathfrak{e}^{(1)} = 0, \quad (68)$$

$$\bar{\mathbb{P}}_{ab}^{(1)} = 0 \Rightarrow \Pi_{ab}^{(1)} = -2\mathcal{K}_{ab} + p^{-1} (\Pi^{(1)} - \mathfrak{e}^{(1)}) h_{ab}. \quad (69)$$

Choosing the isotropy gauge such that  $\Pi^{(1)} = \mathfrak{e}^{(1)} = 0$ , we reach  $\Pi_{ab}^{(1)} = -2\mathcal{K}_{ab}$ .

**ii) Second order.**

With the results in the first order and assuming  $\mathfrak{j}_a^{(2)} = 0$ , we can obtain the second order terms through

$$\bar{\mathbb{H}}^{(2)} = 0 \Rightarrow \mathfrak{e}^{(2)} = -2\mathbb{P}^{-1}\mathcal{K}_{ab}\mathcal{K}^{ab}, \quad (70)$$

$$\bar{\mathbb{P}}_{ab}^{(2)} = 0 \Rightarrow \Pi_{ab}^{(2)} = \mathbb{P}^{-1} [2\mathcal{K}_{ac}\mathcal{K}_b^c - 4\mathcal{K}_{c(a}\Omega_{b)}^c + 4h_a^c h_b^d D\mathcal{K}_{cd}] + p^{-1} (\Pi^{(2)} - \mathfrak{e}^{(2)}) h_{ab}. \quad (71)$$

Choosing the isotropy gauge such that  $\Pi^{(2)} = \mathfrak{e}^{(2)} = -2\mathbb{P}^{-1}\mathcal{K}_{ab}\mathcal{K}^{ab}$ , and employing the derivatives of momentum constraint equation (17) which lead to the identities,

$$\begin{aligned} h_a^c h_b^d D\mathcal{K}_{cd} &= -h_a^c h_b^d \partial_c \partial_d \ln \mathbb{P} - \mathcal{K}_{ab} D \ln \mathbb{P} + D_a^\perp \ln \mathbb{P} D_b^\perp \ln \mathbb{P} - \mathcal{K}_a^c \mathcal{K}_{cb} - \Omega_a^c \Omega_{cb} + O(\partial^3), \\ h^{cd} D\mathcal{K}_{cd} &= D\mathcal{K} = O(\partial^3), \end{aligned} \quad (72)$$

we finally reach the stress tensor up to the second order in the derivative expansion,

$$\bar{T}_{ab} = +\mathbb{P}h_{ab} + (\mathfrak{e}^{(1)} + \mathfrak{e}^{(2)}) u_a u_b + \Pi_{ab}^{(1)} + \Pi_{ab}^{(2)} \quad (73)$$

$$\begin{aligned} &= +\mathbb{P}h_{ab} - 2\mathcal{K}_{ab} - 2\mathbb{P}^{-1} (\mathcal{K}_{ab}\mathcal{K}^{ab}) u_a u_b + \mathbb{P}^{-1} [-2\mathcal{K}_{ac}\mathcal{K}_b^c - 4\mathcal{K}_{c(a}\Omega_{b)}^c \\ &\quad - 4\Omega_{ac}\Omega_b^c - 4h_a^c h_b^d \partial_c \partial_d \ln \mathbb{P} - 4\mathcal{K}_{ab} D \ln \mathbb{P} + 4(D_a^\perp \ln \mathbb{P})(D_b^\perp \ln \mathbb{P})]. \end{aligned} \quad (74)$$

Comparing the above stress tensor  $\bar{T}_{ab}$  with the general stress tensor  $T_{ab}^{(R)}$  in (19), one can read out exactly the same coefficients in (20) when  $\alpha = 0$ . Thus, through using the Hamiltonian constraint and Petrov type I condition, we recover the Brown-York stress tensor (18) dual to the bulk metric in (8) in the case of Einstein gravity.

## 4.2 Recover the Rindler fluid in Einstein-Gauss-Bonnet gravity

In this subsection, we will recover the Rindler fluid in Einstein-Gauss-Bonnet gravity. For the convenience of calculation and since  $\bar{\mathbb{H}} \equiv 0$ , we write the Hamiltonian constraint (50) as

$$\mathbb{H} = \mathbb{H}^{(\alpha)} + \delta\bar{\mathbb{H}}^{(H)} = \delta\mathbb{H}^{(\alpha)} + \delta\bar{\mathbb{H}}^{(H)} = 0, \quad (75)$$

where  $\mathbb{H}^{(\alpha)}$  and  $\delta\bar{\mathbb{H}}^{(H)}$  can be found in (40) and (52), respectively. Since  $\bar{\mathbb{P}}_{ab} \equiv 0$ , the Petrov type I condition in (49) becomes

$$\mathbb{P}_{ab} = \mathbb{P}_{ab}^{(\alpha)} + \delta\bar{\mathbb{P}}_{ab}^{(H)} = \delta\mathbb{P}_{ab}^{(\alpha)} + \delta\bar{\mathbb{P}}_{ab}^{(H)} = 0, \quad (76)$$

where  $\mathbb{P}_{ab}^{(\alpha)}$  and  $\delta\bar{\mathbb{P}}_{ab}^{(H)}$  can be found in (35) and (45). On the other hand, from (56) and with the results in (73), one has

$$2\bar{K}_{ab} = -(\mathbb{P} + \mathfrak{e}^{(2)})u_a u_b - \Pi_{ab}^{(1)} - \Pi_{ab}^{(2)} + O(\partial^3). \quad (77)$$

We then assume the following decomposition of the extrinsic curvature

$$K_{ab} = \varrho u_a u_b + \pi_{ab}, \quad \varrho \equiv K_{ab} u^a u^b, \quad \pi_{ab} \equiv h_a^c h_b^d K_{cd}, \quad (78)$$

$$\delta K_{ab}^{(\alpha)} = \delta\varrho^{(\alpha)} u_a u_b + \delta\pi_{ab}^{(\alpha)}, \quad \delta\varrho^{(\alpha)} \equiv \delta K_{ab}^{(\alpha)} u^a u^b, \quad \delta\pi_{ab}^{(\alpha)} \equiv h_a^c h_b^d \delta K_{cd}^{(\alpha)}. \quad (79)$$

From (46), we then conclude

$$2\varrho = -\mathbb{P} - \mathfrak{e}^{(2)} + 2\delta\varrho^{(\alpha)} + O(\partial^3) + O(\alpha^2), \quad (80)$$

$$2\pi_{ab} = -\Pi_{ab}^{(1)} - \Pi_{ab}^{(2)} + 2\delta\pi_{ab}^{(\alpha)} + O(\partial^3) + O(\alpha^2). \quad (81)$$

Putting (77) into (52) and (35), one has

$$\delta\bar{\mathbb{H}}^{(H)} = O(\partial^3), \quad \delta\bar{\mathbb{P}}_{ab}^{(H)} = -6\alpha\mathbb{P}^2 [\Omega_{ac}\Omega_b^c + p^{-1}h_{ab}\Omega_{cd}\Omega^{cd}] + O(\partial^3). \quad (82)$$

As the Gauss-Bonnet corrections to Hamiltonian constraint and Petrov type I condition appear at the second order in the derivative expansion, we only need to consider the second order corrections that  $\delta\varrho^{(\alpha)} \sim \delta\pi_{ab}^{(\alpha)} \sim O(\partial^2)$ . Thus put (78) into (40) and (35), we have

$$\mathbb{H}^{(\alpha)} = (2\varrho - \pi)\pi + \pi_{ab}\pi^{ab}, \quad (83)$$

$$\mathbb{P}_{ab}^{(\alpha)} = (\pi - 2\varrho)\pi_{ab} - \pi_{ac}\pi_b^c + 2\varrho\mathcal{K}_{ab} + 2\mathcal{K}_{(a}^c\pi_{b)c} + 2\Omega_{(a}^c\pi_{b)c} + 2h_a^c h_b^d D\pi_{cd}. \quad (84)$$

Taking into account of (80) and (81) and consider the first order in the small  $\alpha$  expansion, we obtain

$$\delta\mathbb{H}^{(\alpha)} = \mathbb{H}^{(\alpha)} = -\mathbb{P}\delta\pi^{(\alpha)}, \quad \delta\mathbb{P}_{ab}^{(\alpha)} = \mathbb{P}_{ab}^{(\alpha)} = \mathbb{P}\delta\pi_{ab}^{(\alpha)}. \quad (85)$$

With (82) and (85), at the second order in the derivative expansion, the Hamiltonian constraint leads to

$$\mathbb{H}^{(2)} = \delta\mathbb{H}^{(\alpha)} + \delta\bar{\mathbb{H}}^{(H)} = 0 \Rightarrow \delta\pi^{(\alpha)} = 0. \quad (86)$$

And the Petrov type I condition leads to

$$\mathbb{P}_{ab}^{(2)} = \delta\mathbb{P}_{ab}^{(\alpha)} + \delta\bar{\mathbb{P}}_{ab}^{(H)} = 0 \Rightarrow \delta\pi_{ab}^{(\alpha)} = 6\alpha\mathbb{P} [\Omega_{ac}\Omega_b^c + p^{-1}h_{ab}\Omega_{cd}\Omega^{cd}]. \quad (87)$$

We can see that there is no constraint on  $\varrho^{(\alpha)}$  at this order, and it will be determined by the gauge choice of the stress tensor. Then from (55), we obtain

$$\delta T_{ab}^{(\alpha)} = -2\delta\pi^{(\alpha)}u_a u_b + 2(\delta\pi^{(\alpha)} - \delta\varrho^{(\alpha)})h_{ab} - 2\delta\pi_{ab}^{(\alpha)}. \quad (88)$$

On the other hand, a straightforward calculation from (55) and (77) gives

$$\delta\bar{T}_{ab}^{(J)} = \alpha\mathbb{P} \left[ -\Pi_{ac}^{(1)}\Pi_b^{c(1)} + \frac{1}{2} \left( \Pi_{cd}^{(1)}\Pi_{(1)}^{cd} \right) h_{ab} \right], \quad (89)$$

where  $\Pi_{ab}^{(1)}$  has been obtained in (69). Put them together, we obtain

$$\begin{aligned} \delta T_{ab} = \delta T_{ab}^{(\alpha)} + \delta\bar{T}_{ab}^{(J)} = & -4\alpha\mathbb{P} (\mathcal{K}_{ac}\mathcal{K}_b^c + 3p^{-1}\Omega_{ac}\Omega_b^c) \\ & + [-2\delta\varrho^{(2)} + 2\alpha\mathbb{P} (\mathcal{K}_{cd}\mathcal{K}^{cd} - 6p^{-1}\Omega_{cd}\Omega^{cd})] h_{ab}. \end{aligned} \quad (90)$$

The isotropic gauge of the pressure leads to  $\delta\varrho^{(2)} = \alpha\mathbb{P} (\mathcal{K}_{cd}\mathcal{K}^{cd} - 6p^{-1}\Omega_{cd}\Omega^{cd})$ . Then the stress tensor from Petrov type I condition turns out to be  $\bar{T}_{ab} + \delta T_{ab}$  with (74) and (90), which match exactly with the  $T_{ab}^{(GB)}$  in (18) from the fluid/gravity calculation.

## 5 The non-relativistic hydrodynamic expansion

The Rindler fluid with Gauss-Bonnet corrections in the following non-relativistic hydrodynamic expansion has been studied in [43, 44]

$$v_i \sim \epsilon, \quad P \sim \epsilon^2, \quad \partial_i \sim \epsilon, \quad \partial_\tau \sim \epsilon^2. \quad (91)$$

And the dual tress tensor turns out to be  $\tilde{T}_{ab} = \bar{T}_{ab} + \delta T_{ab}$ , where  $\bar{T}_{ab}$  come from the Einstein sector, which are given by [43],

$$\begin{aligned} \bar{T}_i^\tau &= +r_c^{-3/2}v_i + r_c^{-5/2} [v_i(v^2 + P) - 2r_c\sigma_{ij}v^j] + O(\epsilon^5), \\ \bar{T}_\tau^\tau &= -r_c^{-3/2}v^2 - r_c^{-5/2} [v^2(v^2 + P) - 2r_c\sigma_{ij}v^i v^j - 2r_c^2\sigma_{ij}\sigma^{ij}] + O(\epsilon^6), \\ \bar{T}_{ij} &= +r_c^{-1/2}\delta_{ij} + r_c^{-3/2} [P\delta_{ij} + v_i v_j - 2r_c\sigma_{ij}] \\ &\quad + r_c^{-5/2} [v_i v_j (v^2 + P) - r_c\sigma_{ij}v^2 + 2r_c v_i \partial_j P - r_c v_i \partial_j v^2 - 2r_c^2 v_i \partial^2 v_j] \\ &\quad - 2r_c^2 \sigma_{ik}\sigma^k_j - 4r_c^2 \sigma_{k(i}\omega_{j)}^k - 4r_c^2 \omega_{ik}\omega_j^k - 4r_c^2 \partial_i \partial_j P + 3r_c^3 \partial^2 \sigma_{ij}] + O(\epsilon^6), \\ \bar{T} &= \bar{T}_\tau^\tau + \bar{T}_i^i = pr_c^{-1/2} + pr_c^{-3/2}P + O(\epsilon^6). \end{aligned} \quad (92)$$

Here the fluid shear  $\sigma_{ij} = \partial_{(i}v_{j)}$  and vorticity  $\omega_{ij} = \partial_{[i}v_{j]}$ . And  $\delta T_{ab}$  come from the Gauss-Bonnet term, with the non-vanishing components [44, 47],

$$\delta T_{ij} = -4\alpha r_c^{-3/2} (\sigma_{ik}\sigma^k_j + 3\omega_{ik}\omega^k_j) + O(\epsilon^6), \quad (93)$$

$$\delta T = \delta^{ij}\delta T_{ij} = -4\alpha r_c^{-3/2} (\sigma_{ij}\sigma^{ij} - 3\omega_{ij}\omega^{ij}) + O(\epsilon^6). \quad (94)$$

We can see that the contributions from the Gauss-Bonnet term only appear at order  $\epsilon^4$ . This comes from the fact that the first non-zero components of the Riemann tensor appear at order  $\epsilon^2$  [44]. And notice that the situation for the case of Einstein gravity has been studied in [56]. Thus we need only to focus on the Gauss-Bonnet corrections to the Petrov type I condition and Hamiltonian constraint at  $\epsilon^4$  in this section.

## 5.1 Petrov type I condition in Rindler fluid

Introduce the new coordinate  $x^0 = \sqrt{r_c}\tau$ , the flat induced metric  $\gamma_{ab}$  in (5) becomes

$$ds_{p+1}^2 = \eta_{ab}dx_a dx^b = -(dx^0)^2 + \delta_{ij}dx^i dx^j. \quad (95)$$

The  $(p+2)$  Newman-Penrose-like vector fields are given with respect to the ingoing and outgoing pair of null vectors as [49]

$$\sqrt{2}\ell = \partial_0 - n, \quad \sqrt{2}k = -\partial_0 - n, \quad m_i = \partial_i. \quad (96)$$

Here  $n$  is the unit normal vector of the hypersurface  $\Sigma_c$ ,  $\partial_0$  and  $\partial_i$  are the tangent vectors to  $\Sigma_c$ . The spacetime is at least Petrov type I if

$$P_{ij} \equiv 2C_{(\ell)i(\ell)j} = 0, \quad C_{(\ell)i(\ell)j} \equiv \ell^\mu m_i^\nu \ell^\alpha m_j^\beta C_{\mu\nu\alpha\beta}. \quad (97)$$

With the Guass-Codazzi equations given in (34), we have the Petrov type I condition up to the first order in the small  $\alpha$  expansion as

$$P_{ij} = \bar{P}_{ij} + \delta P_{ij}^{(\alpha)} + \delta \bar{P}_{ij}^{(H)} = 0, \quad (98)$$

$$\bar{P}_{ij} \equiv -\bar{M}_{ij}^\perp + 2\bar{N}_{0ij}^\perp + \bar{M}_{0i0j}^\perp, \quad \delta P_{ij}^{(\alpha)} \equiv -\delta M_{ij}^\perp + 2\delta N_{0ij}^\perp + \delta M_{0i0j}^\perp, \quad (99)$$

with

$$\delta \bar{P}_{ij}^{(H)} = -2\alpha \bar{H}_{ij}^\perp + 2\alpha p^{-1} \delta_{ij} [\bar{H}_{\mu\nu} n^\mu n^\nu - 2\bar{H}_{0\mu} n^\mu + \bar{H}_{00} + \bar{H}] \quad (100)$$

$$\begin{aligned} &= -2\alpha (\bar{M}_i^{cde} \bar{M}_{jcde} + 2\bar{N}_i^{cd} \bar{N}_{jcd} + \bar{N}^{cd}_i \bar{N}_{cdj} + 2\bar{M}_i^d \bar{M}_{jd}) \\ &+ \alpha p^{-1} \delta_{ij} \left[ 2(\bar{M}_0^{cde} \bar{M}_{0cde} + 2\bar{N}_0^{cd} \bar{N}_{0cd} + \bar{N}^{cd}_0 \bar{N}_{cd0} + 2\bar{M}_0^d \bar{M}_{0d}) \right. \\ &\left. + 4(\bar{M}_{0cde} \bar{N}^{dec} - 2\bar{M}^{cd} \bar{N}_{0cd}) + (\bar{M}^{cdef} \bar{M}_{cdef} + 6\bar{N}^{cde} \bar{N}_{cde} + 8\bar{M}^{cd} \bar{M}_{cd}) \right]. \quad (101) \end{aligned}$$

The Hamiltonian constraint becomes

$$H = \bar{H} + \delta H^{(\alpha)} + \delta \bar{H}^{(H)} = 0, \quad (102)$$

$$\bar{H} \equiv \bar{M}, \quad \delta H^{(\alpha)} \equiv \delta M, \quad (103)$$

with

$$\delta\bar{H}^{(H)} \equiv -4\alpha\bar{H}_{\mu\nu}n^\mu n^\nu = \alpha(-4\bar{M}_{ab}\bar{M}^{ab} + \bar{M}_{abcd}\bar{M}^{abcd}). \quad (104)$$

Notice that the frame choice in (96) singles out a preferred time coordinate  $\partial_0$  and thus breaks Lorentz invariance. It has been shown in [56] that with the frame (96), the Petrov type I condition for vacuum Einstein gravity  $\bar{P}_{ij} = 0$  is violated at order  $\epsilon^4$ :

$$\bar{P}_{ij}^{(E)} = \bar{P}_{ij} = \frac{1}{2}r_c^{-3} [6r_c v_k v_{(i} \omega_{j)}^k - 2r_c^2 v_{(i} \partial^2 v_{j)} - 4r_c^2 v^k \partial_{(i} \omega_{j)}^k + r_c^3 \partial^2 \sigma_{ij}] + O(\epsilon^6). \quad (105)$$

However, after straightforward calculations with the stress tensor (92) and (93), we find

$$\delta\bar{H}^{(H)} = \delta H^{(\alpha)} = O(\epsilon^6), \quad (106)$$

$$\delta\bar{P}_{ij}^{(H)} = -\delta P_{ij}^{(\alpha)} = -6\alpha r_c^{-2} (\omega_{ik} \omega_j^k + p^{-1} \delta_{ij} \omega_{kl} \omega^{kl}) + O(\epsilon^5). \quad (107)$$

Thus, there are no Gauss-Bonnet corrections to the Hamiltonian constraint (102) and Petrov type I condition (98) up to order  $\epsilon^4$  and up to the first order in small  $\alpha$ . In the following subsection, we will show that either demand  $\bar{P}_{ij} = 0$  or with the stress tensor (92) of Rindler fluid in vacuum Einstein gravity, and impose

$$\delta H = \delta H^{(\alpha)} + \delta\bar{H}^{(H)} = 0, \quad \delta P_{ij} = \delta P_{ij}^{(\alpha)} + \delta\bar{P}_{ij}^{(H)} = 0, \quad (108)$$

we can get exactly the contribution (93) of the Gauss-Bonnet term to the stress tensor of the dual fluid, without solving the Einstein-Gauss-Bonnet field equations.

## 5.2 Recover the Gauss-Bonnet corrections

If we still demand the Petrov type I condition  $\bar{P}_{ij} = 0$  in the vacuum Einstein gravity, it has been shown in [56] that the stress tensor in (92) can be recovered up to an additional term at  $\epsilon^4$ :

$$\delta\bar{T}_{ij}^{(E)} = r_c^{-5/2} [6r_c v_k v_{(i} \omega_{j)}^k - 2r_c^2 v_{(i} \partial^2 v_{j)} - 4r_c^2 v^k \partial_{(i} \omega_{j)}^k + r_c^3 \partial^2 \sigma_{ij}] + O(\epsilon^6). \quad (109)$$

Then using  $\bar{T}_{ab} + \delta\bar{T}_{ab}^{(E)}$  instead of  $\bar{T}_{ab}$  in (92), we can obtain the extrinsic curvature  $\bar{K}_{ab}$  from (56), and then put them into (104) and (101), which lead to the same results in (106) and (107), we see that

$$\delta\bar{H}^{(H)} = O(\epsilon^6), \quad (110)$$

$$\delta\bar{P}_{ij}^{(H)} = -6\alpha r_c^{-2} (\omega_{ik} \omega_j^k + p^{-1} \delta_{ij} \omega_{kl} \omega^{kl}) + O(\epsilon^5). \quad (111)$$

They are not affected by the additional term  $\delta\bar{T}_{ab}^{(E)}$ . To cancel the non-vanishing  $\delta\bar{P}_{ij}^{(H)}$  at order  $\epsilon^4$  in (111), we assume  $\delta T_{ab} \sim O(\epsilon^4)$  such that  $\delta H^{(\alpha)}$  in (103) and  $\delta P_{ij}^{(\alpha)}$  in (99) also appear at order  $\epsilon^4$ . As  $\bar{T}_i^\tau$  in (92) has been fixed through the frame choice of the



velocity [56], we only need to set the Gauss-Bonnet correction  $\delta T^\tau_i = O(\epsilon^5)$ . Then put the relation (57) into (103) and (99), we obtain

$$\delta H^{(\alpha)} = \frac{1}{2} r_c^{-1/2} [-\delta T^\tau_\tau + 4\alpha (\bar{J} - 3\bar{J}^\tau_\tau)] , \quad (112)$$

$$\delta P^{(\alpha)}_{ij} = \frac{1}{2} r_c^{-1/2} [-\delta T_{ij} - 4\alpha (3\bar{J}_{ij} - 2p^{-1}\bar{J}\delta_{ij}) + p^{-1}\delta T\delta_{ij}] . \quad (113)$$

With (7), (56) and (92), we have the non-zero components of  $\bar{J}_{ab}$  as

$$\bar{J}^\tau_\tau = \frac{1}{6} r_c^{-3/2} (\sigma_{ij}\sigma^{ij}) + O(\epsilon^6), \quad \bar{J}_{ij} = \frac{1}{3} r_c^{-3/2} \sigma_{ik}\sigma^k_j + O(\epsilon^6), \quad \bar{J} = \bar{J}^\tau_\tau + \bar{J}^i_i . \quad (114)$$

Substituting them into (108), we finally obtain

$$\delta T^\tau_\tau = O(\epsilon^6), \quad (115)$$

$$\begin{aligned} \delta T_{ij} = & -4\alpha r_c^{-3/2} (\sigma_{ik}\sigma^k_j + 3\omega_{ik}\omega^k_j) \\ & + p^{-1} [\delta T + 4\alpha r_c^{-3/2} (\sigma_{ij}\sigma^{ij} - 3\omega_{ij}\omega^{ij})] \delta_{ij} + O(\epsilon^6). \end{aligned} \quad (116)$$

After choosing the isotropic gauge such that there are no corrections to the  $\delta_{ij}$  part of the stress tensor at this order as in [43, 44], we have  $\delta T = -4\alpha r_c^{-3/2} (\sigma_{ij}\sigma^{ij} - 3\omega_{ij}\omega^{ij})$ . These results exactly match with the Gauss-Bonnet corrections in the stress tensor of Rindler fluid which are given in (93) and (94) from the fluid/gravity calculation.

Alternatively, once the stress tensor  $\bar{T}_{ab}$  of Rindler fluid is given in vacuum Einstein gravity (92) from the fluid/gravity calculation, by demanding the condition (108) to hold that the additional Gauss-Bonnet corrections to the Hamiltonian constraint and Petrov type I condition vanish, one can show that the formulas between (110) and (116) are the same as the those in the case by using  $\bar{T}_{ab} + \delta\bar{T}_{ab}^{(E)}$ , such that we can again obtain the Gauss-Bonnet corrections to the stress tensor of the Rindler fluid in (93) for the Einstein-Gauss-Bonnet gravity.

## 6 Conclusion

To summarize, we have checked the Petrov type I condition for the vacuum solutions of Einstein-Gauss-Bonnet gravity in both relativistic and non-relativistic hydrodynamic expansions. With the solution constructed in [47], we have shown that the spacetime is at least Petrov type I up to the second order in the relativistic hydrodynamic expansion. Turn the logic around, assuming the Hamiltonian constraint and Petrov type I condition on a finite cutoff hypersurface, we have shown that the dual stress tensor can be recovered with correct first and second order transport coefficients by taking the Gauss-Bonnet coefficient as an expansion parameter. While in the non-relativistic hydrodynamic expansion [44], although the Petrov type I condition is violated at order  $\epsilon^4$  in the vacuum Einstein gravity [56], we have found that the Gauss-Bonnet term does not contribute to the violation terms in the Petrov type I condition up to  $\epsilon^4$ . Thus, given the stress tensor of the Rindler fluid

in vacuum Einstein gravity, we have shown that demanding the additional Gauss-Bonnet corrections to the Petrov type I condition and Hamiltonian constraint vanish at the first order of  $\alpha$  expansion, the Gauss-Bonnet corrections to the stress tensor of dual fluid can also be recovered.

Notice that in both cases, in order to recover the stress tensor of dual fluid from the Petrov type I condition, we have additionally taken the small  $\alpha$  limit. And up to the first order of  $\alpha$  expansion, the Petrov type I condition can be expressed as a function of extrinsic curvature and other intrinsic quantities on the hypersurface. Actually, note the fact that the Einstein-Gauss-Bonnet field equations are quasi-linear in terms of  $\alpha$  [61, 60], and the dual stress tensor with Gauss-Bonnet corrections in (18) is also linear in terms of  $\alpha$ . It is not surprised that we can still recover the stress tensor (18) even when we take the small  $\alpha$  limit.

So far most of studies on the Petrov type I condition has been focused on the case with asymptotically flat spacetimes. It is quite important and interesting to investigate corresponding ones for asymptotically AdS spacetimes based on the AdS/CFT correspondence, as the regularity condition on the future horizon of spacetime is necessary and important for the perturbations in the fluid/gravity correspondence and imposing the Petrov type I condition on the spacetime is mathematically much simpler than directly solving the perturbative gravitational field equations in order to find the stress tensor of dual fluid. On the other hand, the KSS bound [4] states that the universal value of the ratio of shear viscosity over entropy density from the AdS/CFT calculation is always above  $\eta/s = 1/4\pi$ , while in the AdS gravity with curvature squared corrections, the bound is found to be violated by the Gauss-Bonnet term [62, 63, 64]. With the static black brane solution in [65], it is expected that the universal value with Gauss-Bonnet correction  $\eta/s = [1 - 2(p+1)(p-2)\alpha]/4\pi$  can also be recovered from the Petrov type I condition on the dual fluid.

## Acknowledgments

This work is supported by National Natural Science Foundation of China (No.10821504, No.11035008, and No.11375247). We thank L. Li for helpful conversation. Y. L. Zhang thanks Professors K. Skenderis and M. Taylor for many valuable discussions on this topic, as well as C. Eling and A. Meyer for helpful correspondence.

## A Classification of the Weyl tensor

In four dimensional spacetime, tensor classification plays an important role in studying the exact solutions of Einstein field equations [66]. And in particular, the Petrov type classification of Weyl tensor has interesting physical applications. It has been generalized to the arbitrarily higher dimensional spacetimes in [67]. In this appendix, we briefly summarize these results based on [68, 69], which can also be reduced to the Petrov type classification in four dimensions.

Consider a  $p + 2$  dimensional Lorentz manifold ( $p \geq 2$ ) with signature  $(- + \dots +)$  and choose a null frame  $\ell, \mathbf{k}, \mathbf{m}_i$ , which satisfies the following orthogonal and normalization conditions

$$\ell^2 = \mathbf{k}^2 = 0, \quad (\mathbf{k}, \ell) = 1, \quad (\mathbf{m}_i, \mathbf{k}) = (\mathbf{m}_i, \ell) = 0, \quad (\mathbf{m}_i, \mathbf{m}_j) = \delta_{ij}, \quad (117)$$

so that in this frame the metric of the manifold can be decomposed as

$$g_{\mu\nu} = 2\ell_{(\mu}\mathbf{k}_{\nu)} + \delta_{ij}\mathbf{m}_\mu^i\mathbf{m}_\nu^j, \quad g^{\mu\nu} = 2\ell^{(\mu}\mathbf{k}^{\nu)} + \delta^{ij}\mathbf{m}_i^\mu\mathbf{m}_j^\nu. \quad (118)$$

The null frame is covariant under the following boost transformation,

$$\ell \rightarrow \lambda \ell, \quad \mathbf{k} \rightarrow \lambda^{-1}\mathbf{k}, \quad \mathbf{m}_i \rightarrow \mathbf{m}_i, \quad \lambda \neq 0. \quad (119)$$

For a rank  $q$  tensor  $T$  on the manifold, its components  $T_{\mu_1 \dots \mu_q}$  with fixed list of indices are null frame scalars, and they transform under the boost transformation as

$$T_{\mu_1 \dots \mu_q} \rightarrow \lambda^{b_{\{\mu\}}} T_{\mu_1 \dots \mu_q}, \quad b_{\{\mu\}} = b_{\mu_1} + \dots + b_{\mu_q}, \quad b_{(\ell)} = 1, \quad b_i = 0, \quad b_{(\mathbf{k})} = -1. \quad (120)$$

$b$  is named as the boost-weight of the null-frame scalar  $T_{\mu_1 \dots \mu_q}$ . The boost order (along  $\ell$ ) of the tensor  $T$  is defined to be the largest value of  $b_{\{\mu\}}$  among all the non-vanishing components  $T_{\mu_1 \dots \mu_q}$ . It is only a function of the null direction  $\ell$  and is denoted as  $\mathcal{B}(\ell)$ .

The Weyl tensor can be decomposed and sorted by the boost weight of its components,

$$C_{\alpha\beta\gamma\delta} = C_{\alpha\beta\gamma\delta}^{[2]} + C_{\alpha\beta\gamma\delta}^{[1]} + C_{\alpha\beta\gamma\delta}^{[0]} + C_{\alpha\beta\gamma\delta}^{[-1]} + C_{\alpha\beta\gamma\delta}^{[-2]}, \quad (121)$$

where the superscript index indicates the boost weight and

$$\begin{aligned} C_{\alpha\beta\gamma\delta}^{[2]} &= 4C_{(\ell)i(\ell)j}\mathbf{k}_{\{\alpha}\mathbf{m}_{\beta}^i\mathbf{k}_{\gamma}\mathbf{m}_{\delta}^j\}, \\ C_{\alpha\beta\gamma\delta}^{[1]} &= 8C_{(\ell)(\mathbf{k})(\ell)i}\mathbf{k}_{\{\alpha}\mathbf{l}_{\beta}\mathbf{k}_{\gamma}\mathbf{m}_{\delta}^i\} + 4C_{(\ell)ijk}\mathbf{k}_{\{\alpha}\mathbf{m}_{\beta}^i\mathbf{m}_{\gamma}^j\mathbf{m}_{\delta}^k\}, \\ C_{\alpha\beta\gamma\delta}^{[0]} &= 4C_{(\ell)(\mathbf{k})(\ell)(\mathbf{k})}\mathbf{k}_{\{\alpha}\mathbf{l}_{\beta}\mathbf{k}_{\gamma}\mathbf{l}_{\delta}\} + 4C_{(\ell)(\mathbf{k})ij}\mathbf{k}_{\{\alpha}\mathbf{l}_{\beta}\mathbf{m}_{\gamma}^i\mathbf{m}_{\delta}^j\} \\ &\quad + 8C_{(\ell)i(\mathbf{k})j}\mathbf{k}_{\{\alpha}\mathbf{m}_{\beta}^i\mathbf{l}_{\gamma}\mathbf{m}_{\delta}^j\} + C_{ijkl}\mathbf{m}_{\{\alpha}^i\mathbf{m}_{\beta}^j\mathbf{m}_{\gamma}^k\mathbf{m}_{\delta}^l\}, \\ C_{\alpha\beta\gamma\delta}^{[-1]} &= 8C_{(\mathbf{k})(\ell)(\mathbf{k})i}\mathbf{l}_{\{\alpha}\mathbf{k}_{\beta}\mathbf{l}_{\gamma}\mathbf{m}_{\delta}^i\} + 4C_{(\mathbf{k})ijk}\mathbf{l}_{\{\alpha}\mathbf{m}_{\beta}^i\mathbf{m}_{\gamma}^j\mathbf{m}_{\delta}^k\}, \\ C_{\alpha\beta\gamma\delta}^{[-2]} &= 4C_{(\mathbf{k})i(\mathbf{k})j}\mathbf{l}_{\{\alpha}\mathbf{m}_{\beta}^i\mathbf{l}_{\gamma}\mathbf{m}_{\delta}^j\}. \end{aligned} \quad (122)$$

The notations  $T_{\{\alpha\beta\gamma\delta\}} \equiv (T_{[\alpha\beta][\gamma\delta]} + T_{[\gamma\delta][\alpha\beta]})/2$ , as well as  $C_{(\ell)i(\mathbf{k})j} \equiv C_{(\ell)i(\ell)j}\ell^\mu\mathbf{m}_j^\alpha\mathbf{k}^\nu\mathbf{m}_j^\beta$  and so on, have been introduced. The Weyl tensor is generically of boost order  $\mathcal{B}(\ell) = 2$ , and a null vector  $\ell$  is defined to be aligned with the Weyl tensor whenever  $\mathcal{B}(\ell) \leq 1$ . In this case,  $\ell$  is a Weyl aligned null direction, and  $1 - \mathcal{B}(\ell) \in \{0, 1, 2, 3\}$  is the order of alignment. It usually depends on the rank and symmetry properties of the tensors.

According to [67], the principal type of the Weyl tensor in a Lorentzian manifold is I, II, III, N according to whether there exists an aligned  $\ell$  of alignment order 0, 1, 2, 3, respectively. If no aligned  $\ell$  exists, the manifold is of (general) type G, if the Weyl tensor

vanishes the manifold is of type O. The algebraically special types with necessary condition are summarized as follows:

$$\begin{aligned}
\text{Type I : } & C_{(\ell)i(\ell)j} = 0, \\
\text{Type II : } & C_{(\ell)i(\ell)j} = C_{(\ell)ijk} = 0, \\
\text{Type III : } & C_{(\ell)i(\ell)j} = C_{(\ell)ijk} = C_{ijkl} = C_{(\ell)(\mathbf{k})ij} = 0, \\
\text{Type N : } & C_{(\ell)i(\ell)j} = C_{(\ell)ijk} = C_{ijkl} = C_{(\ell)(\mathbf{k})ij} = C_{(\mathbf{k})ijk} = 0.
\end{aligned} \tag{123}$$

Following the curvature tensor symmetries and the trace-free condition [68], one can reach some familiar Petrov types with the following properties,

$$\begin{aligned}
\text{Type I : } & C_{\alpha\beta\gamma\delta}^{[2]} = 0, \\
\text{Type II : } & C_{\alpha\beta\gamma\delta}^{[2]} = C_{\alpha\beta\gamma\delta}^{[1]} = 0, \\
\text{Type D : } & C_{\alpha\beta\gamma\delta}^{[2]} = C_{\alpha\beta\gamma\delta}^{[1]} = C_{\alpha\beta\gamma\delta}^{[-1]} = C_{\alpha\beta\gamma\delta}^{[-2]} = 0, \\
\text{Type III : } & C_{\alpha\beta\gamma\delta}^{[2]} = C_{\alpha\beta\gamma\delta}^{[1]} = C_{\alpha\beta\gamma\delta}^{[0]} = 0, \\
\text{Type N : } & C_{\alpha\beta\gamma\delta}^{[2]} = C_{\alpha\beta\gamma\delta}^{[1]} = C_{\alpha\beta\gamma\delta}^{[0]} = C_{\alpha\beta\gamma\delta}^{[-1]} = 0, \\
\text{Type O : } & C_{\alpha\beta\gamma\delta}^{[2]} = C_{\alpha\beta\gamma\delta}^{[1]} = C_{\alpha\beta\gamma\delta}^{[0]} = C_{\alpha\beta\gamma\delta}^{[-1]} = C_{\alpha\beta\gamma\delta}^{[-2]} = 0.
\end{aligned} \tag{124}$$

Further classifications in more detail can be found in [68, 69].

## B Decomposition of the Riemann tensor

The Riemann tensor and its contractions can be decomposed along and perpendicular to a spacelike unit normal vector  $n$ ,

$$\begin{aligned}
g_\mu^\alpha g_\nu^\beta g_\sigma^\gamma g_\lambda^\delta R_{\alpha\beta\gamma\delta} &= M_{\mu\nu\sigma\lambda} - n_\mu N_{\sigma\lambda\nu} + n_\nu N_{\sigma\lambda\mu} - n_\sigma N_{\mu\nu\lambda} + n_\lambda N_{\mu\nu\sigma} \\
&\quad + n_\mu n_\sigma Y_{\nu\lambda} - n_\mu n_\lambda Y_{\nu\sigma} + n_\nu n_\lambda Y_{\mu\sigma} - n_\nu n_\sigma Y_{\mu\lambda}, \\
g_\mu^\alpha g_\nu^\beta R_{\alpha\beta} &= M_{\mu\nu} + n_\mu N_\nu + n_\nu N_\mu + Y_{\mu\nu} + n_\mu n_\nu Y, \\
R &= M + 2Y = -M + 2\gamma^{\beta\delta} R_{\beta\delta},
\end{aligned} \tag{125}$$

where we have defined the following notations with  $\gamma_{\mu\nu} = g_{\mu\nu} - n_\mu n_\nu$ ,

$$\begin{aligned}
M_{\mu\nu\sigma\lambda} &\equiv \gamma_\mu^\alpha \gamma_\nu^\beta \gamma_\sigma^\gamma \gamma_\lambda^\delta R_{\alpha\beta\gamma\delta}, & N_{\mu\nu\sigma} &\equiv \gamma_\mu^\alpha \gamma_\nu^\beta \gamma_\sigma^\gamma n^\delta R_{\alpha\beta\gamma\delta}, & Y_{\mu\nu} &\equiv \gamma_\mu^\alpha n^\beta \gamma_\nu^\gamma n^\delta R_{\alpha\beta\gamma\delta}, \\
M_{\mu\nu} &\equiv \gamma^{\alpha\beta} M_{\mu\alpha\nu\beta}, & M &\equiv \gamma^{\alpha\beta} M_{\alpha\beta}, & N_\mu &\equiv \gamma^{\alpha\beta} N_{\alpha\mu\beta}, & Y &\equiv \gamma^{\alpha\beta} Y_{\alpha\beta}.
\end{aligned} \tag{126}$$

One can also obtain the decomposition of their combinations, such as,

$$\begin{aligned}
R_\mu^{\sigma\lambda\rho} R_{\nu\sigma\lambda\rho} n^\mu n^\nu &= N^{cde} N_{cde} + 2Y^{cd} Y_{cd}, \\
R_\mu^{\sigma\lambda\rho} R_{\nu\sigma\lambda\rho} n^\mu h_b^\nu &= -M_{bcde} N^{dec} - 2Y^{cd} N_{bcd}, \\
R_\mu^{\sigma\lambda\rho} R_{\nu\sigma\lambda\rho} h_a^\mu h_b^\nu &= M_a^{cde} M_{bcde} + 2N_a^{cd} N_{bcd} + N_a^{cd} N_{cdb} + 2Y_a^c Y_{cb},
\end{aligned}$$

$$R_\mu^{\sigma\lambda\rho} R_{\nu\sigma\lambda\rho} g^{\mu\nu} = M^{cdef} M_{cdef} + 4N^{cde} N_{cde} + 4Y^{cd} Y_{cd}. \quad (127)$$

Then  $\bar{H}_{\mu\nu} \equiv \bar{R}_\mu^{\sigma\lambda\rho} \bar{R}_{\nu\sigma\lambda\rho} - \frac{1}{4} (\bar{R}^{\kappa\sigma\lambda\rho} \bar{R}_{\kappa\sigma\lambda\rho}) \bar{g}_{\mu\nu}$  in (43) can be decomposed as

$$\begin{aligned} \bar{H}_{(n)(n)} &\equiv \bar{H}_{\mu\nu} n^\mu n^\nu = \bar{Y}^{cd} \bar{Y}_{cd} - \frac{1}{4} \bar{M}^{cdef} \bar{M}_{cdef}, \\ \bar{H}_{(n)(u)} &\equiv \bar{H}_{\mu\nu} n^\mu \gamma_b^\nu u^b = -\bar{M}_{(u)cde} \bar{N}^{dec} - 2\bar{Y}^{cd} \bar{N}_{(u)cd}, \\ \bar{H}_{(u)(u)} &\equiv \bar{H}_{\mu\nu} \gamma_a^\mu \gamma_b^\nu u^a u^b = \bar{M}_{(u)}^{cde} \bar{M}_{(u)cde} + 2\bar{N}_{(u)}^{cd} \bar{N}_{(u)cd} + \bar{N}_{(u)}^{cd} \bar{N}_{cd(u)} + 2\bar{Y}_{(u)}^d \bar{Y}_{(u)d} \\ &\quad + \frac{1}{4} (\bar{M}^{cdef} \bar{M}_{cdef} + 4\bar{N}^{cde} \bar{N}_{cde} + 4\bar{Y}^{cd} \bar{Y}_{cd}), \\ \bar{H}_{ab}^\perp &\equiv \bar{H}_{\mu\nu} \gamma_c^\mu \gamma_d^\nu h_a^c h_b^d = h_a^m h_b^n (\bar{M}_m^{cde} \bar{M}_{ncde} + 2\bar{N}_m^{cd} \bar{N}_{ncd} + \bar{N}_m^{cd} \bar{N}_{cdn} + 2\bar{Y}_m^d \bar{Y}_{nd}) \\ &\quad - \frac{1}{4} (\bar{M}^{cdef} \bar{M}_{cdef} + 4\bar{N}^{cde} \bar{N}_{cde} + 4\bar{Y}^{cd} \bar{Y}_{cd}) h_{ab}, \\ \bar{H} &\equiv \bar{H}_{\mu\nu} g^{\mu\nu} = -\frac{p-2}{4} (\bar{M}^{cdef} \bar{M}_{cdef} + 4\bar{N}^{cde} \bar{N}_{cde} + 4\bar{Y}^{cd} \bar{Y}_{cd}). \end{aligned} \quad (128)$$

## References

- [1] T. Damour, (1979), Quelques propriétés mécaniques, électromagnétiques, thermodynamiques et quantiques des trous noirs, Thèse de doctorat d'État, Université Paris 6. [T. Damour](#), (1982), Surface effects in black hole physics, in Proceedings of the Second Marcel Grossmann Meeting on General Relativity, Ed. R. Ruffini, North Holland, p. 587.
- [2] T. Damour, “Black Hole Eddy Currents,” [Phys. Rev. D](#) **18**, 3598 (1978).
- [3] R. H. Price and K. S. Thorne, “Membrane Viewpoint On Black Holes: Properties And Evolution Of The Stretched Horizon,” [Phys. Rev. D](#) **33**, 915 (1986).
- [4] P. Kovtun, D. T. Son and A. O. Starinets, “Holography and hydrodynamics: Diffusion on stretched horizons,” [JHEP](#) **0310**, 064 (2003) [[hep-th/0309213](#)].
- [5] E. Gourgoulhon and J. L. Jaramillo, “A 3+1 perspective on null hypersurfaces and isolated horizons,” [Phys. Rept.](#) **423**, 159 (2006) [[gr-qc/0503113](#)].
- [6] E. Gourgoulhon, “A Generalized Damour-Navier-Stokes equation applied to trapping horizons,” [Phys. Rev. D](#) **72**, 104007 (2005) [[gr-qc/0508003](#)].
- [7] C. Eling, I. Fouxon and Y. Oz, “The Incompressible Navier-Stokes Equations From Membrane Dynamics,” [Phys. Lett. B](#) **680**, 496 (2009) [[arXiv:0905.3638 \[hep-th\]](#)].
- [8] C. Eling and Y. Oz, “Relativistic CFT Hydrodynamics from the Membrane Paradigm,” [JHEP](#) **1002**, 069 (2010) [[arXiv:0906.4999 \[hep-th\]](#)].
- [9] J. M. Maldacena, “The Large N limit of superconformal field theories and supergravity,” [Adv. Theor. Math. Phys.](#) **2**, 231 (1998) [[hep-th/9711200](#)].

- [10] S. S. Gubser, I. R. Klebanov and A. M. Polyakov, “Gauge theory correlators from noncritical string theory,” *Phys. Lett. B* **428**, 105 (1998) [[hep-th/9802109](#)].
- [11] E. Witten, “Anti-de Sitter space and holography,” *Adv. Theor. Math. Phys.* **2**, 253 (1998) [[hep-th/9802150](#)].
- [12] O. Aharony, S. S. Gubser, J. M. Maldacena, H. Ooguri and Y. Oz, “Large N field theories, string theory and gravity,” *Phys. Rept.* **323**, 183 (2000) [[hep-th/9905111](#)].
- [13] G. Policastro, D. T. Son and A. O. Starinets, “The Shear viscosity of strongly coupled N=4 supersymmetric Yang-Mills plasma,” *Phys. Rev. Lett.* **87**, 081601 (2001) [[hep-th/0104066](#)].
- [14] G. Policastro, D. T. Son and A. O. Starinets, “From AdS / CFT correspondence to hydrodynamics,” *JHEP* **0209**, 043 (2002) [[hep-th/0205052](#)].
- [15] G. Policastro, D. T. Son and A. O. Starinets, “From AdS / CFT correspondence to hydrodynamics. 2. Sound waves,” *JHEP* **0212**, 054 (2002) [[hep-th/0210220](#)].
- [16] S. Bhattacharyya, V. E. Hubeny, S. Minwalla and M. Rangamani, “Nonlinear Fluid Dynamics from Gravity,” *JHEP* **0802**, 045 (2008) [[arXiv:0712.2456](#) [[hep-th](#)]].
- [17] M. Van Raamsdonk, “Black Hole Dynamics From Atmospheric Science,” *JHEP* **0805**, 106 (2008) [[arXiv:0802.3224](#) [[hep-th](#)]].
- [18] M. Haack and A. Yarom, “Nonlinear viscous hydrodynamics in various dimensions using AdS/CFT,” *JHEP* **0810**, 063 (2008) [[arXiv:0806.4602](#) [[hep-th](#)]].
- [19] S. Bhattacharyya, R. Loganayagam, I. Mandal, S. Minwalla and A. Sharma, “Conformal Nonlinear Fluid Dynamics from Gravity in Arbitrary Dimensions,” *JHEP* **0812**, 116 (2008) [[arXiv:0809.4272](#) [[hep-th](#)]].
- [20] S. Bhattacharyya, S. Minwalla and S. R. Wadia, “The Incompressible Non-Relativistic Navier-Stokes Equation from Gravity,” *JHEP* **0908**, 059 (2009) [[arXiv:0810.1545](#) [[hep-th](#)]].
- [21] N. Iqbal and H. Liu, “Universality of the hydrodynamic limit in AdS/CFT and the membrane paradigm,” *Phys. Rev. D* **79**, 025023 (2009) [[arXiv:0809.3808](#) [[hep-th](#)]].
- [22] D. Nickel and D. T. Son, “Deconstructing holographic liquids,” *New J. Phys.* **13**, 075010 (2011) [[arXiv:1009.3094](#) [[hep-th](#)]].
- [23] I. Bredberg, C. Keeler, V. Lysov and A. Strominger, “Wilsonian Approach to Fluid/Gravity Duality,” *JHEP* **1103**, 141 (2011) [[arXiv:1006.1902](#) [[hep-th](#)]].
- [24] I. Bredberg, C. Keeler, V. Lysov and A. Strominger, “From Navier-Stokes To Einstein,” *JHEP* **1207**, 146 (2012) [[arXiv:1101.2451](#) [[hep-th](#)]].
- [25] I. Bredberg and A. Strominger, “Black Holes as Incompressible Fluids on the Sphere,” *JHEP* **1205**, 043 (2012) [[arXiv:1106.3084](#) [[hep-th](#)]].

- [26] D. Anninos, T. Anous, I. Bredberg and G. S. Ng, “Incompressible Fluids of the de Sitter Horizon and Beyond,” *JHEP* **1205**, 107 (2012) [[arXiv:1110.3792 \[hep-th\]](#)].
- [27] R. -G. Cai, L. Li and Y. -L. Zhang, “Non-Relativistic Fluid Dual to Asymptotically AdS Gravity at Finite Cutoff Surface,” *JHEP* **1107**, 027 (2011) [[arXiv:1104.3281 \[hep-th\]](#)].
- [28] S. Kuperstein and A. Mukhopadhyay, “The unconditional RG flow of the relativistic holographic fluid,” *JHEP* **1111**, 130 (2011) [[arXiv:1105.4530 \[hep-th\]](#)].
- [29] D. Brattán, J. Camps, R. Loganayagam and M. Rangamani, “CFT dual of the AdS Dirichlet problem : Fluid/Gravity on cut-off surfaces,” *JHEP* **1112**, 090 (2011) [[arXiv:1106.2577 \[hep-th\]](#)].
- [30] C. Niu, Y. Tian, X. N. Wu and Y. Ling, “Incompressible Navier-Stokes Equation from Einstein-Maxwell and Gauss-Bonnet-Maxwell Theories,” *Phys. Lett. B* **711**, 411 (2012) [[arXiv:1107.1430 \[hep-th\]](#)].
- [31] X. Bai, Y. P. Hu, B. H. Lee and Y. L. Zhang, “Holographic Charged Fluid with Anomalous Current at Finite Cutoff Surface in Einstein-Maxwell Gravity,” *JHEP* **1211**, 054 (2012) [[arXiv:1207.5309 \[hep-th\]](#)].
- [32] R. G. Cai, T. J. Li, Y. H. Qi and Y. L. Zhang, “Incompressible Navier-Stokes Equations from Einstein Gravity with Chern-Simons Term,” *Phys. Rev. D* **86**, 086008 (2012) [[arXiv:1208.0658 \[hep-th\]](#)].
- [33] D. C. Zou, S. J. Zhang and B. Wang, “Holographic charged fluid dual to third order Lovelock gravity,” *Phys. Rev. D* **87**, 084032 (2013) [[arXiv:1302.0904 \[hep-th\]](#)].
- [34] R. Emparan, V. E. Hubeny and M. Rangamani, “Effective hydrodynamics of black D3-branes,” *JHEP* **1306**, 035 (2013) [[arXiv:1303.3563 \[hep-th\]](#)].
- [35] D. C. Zou and B. Wang, “Holographic parity violating charged fluid dual to Chern-Simons modified gravity,” *Phys. Rev. D* **89**, 064036 (2014) [[arXiv:1306.5486 \[hep-th\]](#)].
- [36] C. Eling and Y. Oz, “Holographic Screens and Transport Coefficients in the Fluid/Gravity Correspondence,” *Phys. Rev. Lett.* **107**, 201602 (2011) [[arXiv:1107.2134 \[hep-th\]](#)].
- [37] R. -G. Cai, L. Li, Z. -Y. Nie and Y. -L. Zhang, “Holographic Forced Fluid Dynamics in Non-relativistic Limit,” *Nucl. Phys. B* **864**, 260 (2012) [[arXiv:arXiv:1202.4091 \[hep-th\]](#)].
- [38] Y. Matsuo, M. Natsuume, M. Ohta and T. Okamura, “The Incompressible Rindler fluid versus the Schwarzschild-AdS fluid,” *PTEP* **2013**, 023B01 (2013) [[arXiv:1206.6924 \[hep-th\]](#)].
- [39] S. Kuperstein and A. Mukhopadhyay, “Spacetime emergence via holographic RG flow from incompressible Navier-Stokes at the horizon,” *JHEP* **1311**, 086 (2013) [[arXiv:1307.1367 \[hep-th\]](#)].
- [40] N. Pinzani-Fokeeva and M. Taylor, “Towards a general fluid/gravity correspondence,” [[arXiv:1401.5975 \[hep-th\]](#)].



- [41] M. M. Caldarelli, J. Camps, B. Gouteraux and K. Skenderis, “AdS/Ricci-flat correspondence and the Gregory-Laflamme instability,” *Phys. Rev. D* **87**, 061502 (2013) [[arXiv:1211.2815 \[hep-th\]](#)].
- [42] M. M. Caldarelli, J. Camps, B. Gout  raux and K. Skenderis, “AdS/Ricci-flat correspondence,” *JHEP* **1404**, 071 (2014) [[arXiv:1312.7874 \[hep-th\]](#)].
- [43] G. Compere, P. McFadden, K. Skenderis and M. Taylor, “The Holographic fluid dual to vacuum Einstein gravity,” *JHEP* **1107**, 050 (2011) [[arXiv:1103.3022 \[hep-th\]](#)].
- [44] G. Chirco, C. Eling and S. Liberati, “Higher Curvature Gravity and the Holographic fluid dual to flat spacetime,” *JHEP* **1108**, 009 (2011) [[arXiv:1105.4482 \[hep-th\]](#)].
- [45] G. Compere, P. McFadden, K. Skenderis and M. Taylor, “The relativistic fluid dual to vacuum Einstein gravity,” *JHEP* **1203**, 076 (2012) [[arXiv:1201.2678 \[hep-th\]](#)].
- [46] C. Eling, A. Meyer and Y. Oz, “The Relativistic Rindler Hydrodynamics,” *JHEP* **1205**, 116 (2012) [[arXiv:1201.2705 \[hep-th\]](#)].
- [47] C. Eling, A. Meyer and Y. Oz, “Local Entropy Current in Higher Curvature Gravity and Rindler Hydrodynamics,” *JHEP* **1208**, 088 (2012) [[arXiv:1205.4249 \[hep-th\]](#)].
- [48] A. Meyer and Y. Oz, “Constraints on Rindler Hydrodynamics,” *JHEP* **1307**, 090 (2013) [[arXiv:1304.6305](#)].
- [49] V. Lysov and A. Strominger, “From Petrov-Einstein to Navier-Stokes,” [arXiv:1104.5502 \[hep-th\]](#).
- [50] T. -Z. Huang, Y. Ling, W. -J. Pan, Y. Tian and X. -N. Wu, “From Petrov-Einstein to Navier-Stokes in Spatially Curved Spacetime,” *JHEP* **1110**, 079 (2011) [[arXiv:1107.1464 \[gr-qc\]](#)].
- [51] T. -Z. Huang, Y. Ling, W. -J. Pan, Y. Tian and X. -N. Wu, “Fluid/gravity duality with Petrov-like boundary condition in a spacetime with a cosmological constant,” *Phys. Rev. D* **85**, 123531 (2012) [[arXiv:1111.1576 \[hep-th\]](#)].
- [52] C. -Y. Zhang, Y. Ling, C. Niu, Y. Tian and X. -N. Wu, “Magnetohydrodynamics from gravity,” *Phys. Rev. D* **86**, 084043 (2012) [[arXiv:1204.0959 \[hep-th\]](#)].
- [53] X. Wu, Y. Ling, Y. Tian and C. Zhang, “Fluid/Gravity Correspondence for General Non-rotating Black Holes,” *Class. Quant. Grav.* **30**, 145012 (2013) [[arXiv:1303.3736 \[hep-th\]](#)].
- [54] B. Wu and L. Zhao, “Gravity-mediated holography in fluid dynamics,” *Nucl. Phys. B* **874**, 177 (2013) [[arXiv:1303.4475 \[hep-th\]](#)].
- [55] Y. Ling, C. Niu, Y. Tian, X. -N. Wu and W. Zhang, “A note on the Petrov-like boundary condition at finite cutoff surface in Gravity/Fluid duality,” [arXiv:1306.5633 \[gr-qc\]](#).
- [56] R. -G. Cai, L. Li, Q. Yang and Y. -L. Zhang, “Petrov type I Condition and Dual Fluid Dynamics,” *JHEP* **1304**, 118 (2013) [[arXiv:1302.2016 \[hep-th\]](#)].



- [57] R. G. Cai, Q. Yang and Y. L. Zhang, “Petrov type I Spacetime and Dual Relativistic Fluids,” *Phys. Rev. D* **90**, 041901 (2014) [[arXiv:1401.7792 \[hep-th\]](#)].
- [58] R. C. Myers, “Higher Derivative Gravity, Surface Terms and String Theory,” *Phys. Rev. D* **36**, 392 (1987).
- [59] S. C. Davis, “Generalized Israel junction conditions for a Gauss-Bonnet brane world,” *Phys. Rev. D* **67**, 024030 (2003) [[hep-th/0208205](#)].
- [60] K. -i. Maeda and T. Torii, “Covariant gravitational equations on brane world with Gauss-Bonnet term,” *Phys. Rev. D* **69**, 024002 (2004) [[hep-th/0309152](#)].
- [61] N. Deruelle and J. Madore, “On the quasilinearity of the Einstein-’Gauss-Bonnet’ gravity field equations,” [gr-qc/0305004](#).
- [62] Y. Kats and P. Petrov, “Effect of curvature squared corrections in AdS on the viscosity of the dual gauge theory,” *JHEP* **0901**, 044 (2009) [[arXiv:0712.0743 \[hep-th\]](#)].
- [63] M. Brigante, H. Liu, R. C. Myers, S. Shenker and S. Yaida, “Viscosity Bound Violation in Higher Derivative Gravity,” *Phys. Rev. D* **77**, 126006 (2008) [[arXiv:0712.0805 \[hep-th\]](#)].
- [64] M. Brigante, H. Liu, R. C. Myers, S. Shenker and S. Yaida, “The Viscosity Bound and Causality Violation,” *Phys. Rev. Lett.* **100**, 191601 (2008) [[arXiv:0802.3318 \[hep-th\]](#)].
- [65] R. -G. Cai, “Gauss-Bonnet black holes in AdS spaces,” *Phys. Rev. D* **65**, 084014 (2002) [[hep-th/0109133](#)].
- [66] H. Stephani, D. Kramer, M. A. H. MacCallum, C. Hoenselaers and E. Herlt, “Exact solutions of Einstein’s field equations,” *Cambridge, UK: Univ. Pr.* (2003) 701 P
- [67] R. Milson, A. Coley, V. Pravda and A. Pravdova, “Alignment and algebraically special tensors in Lorentzian geometry,” *Int. J. Geom. Meth. Mod. Phys.* **2**, 41 (2005) [[gr-qc/0401010](#)].
- [68] A. Coley, R. Milson, V. Pravda and A. Pravdova, “Classification of the Weyl tensor in higher dimensions,” *Class. Quant. Grav.* **21**, L35 (2004) [[gr-qc/0401008](#)].
- [69] A. Coley, “Classification of the Weyl Tensor in Higher Dimensions and Applications,” *Class. Quant. Grav.* **25**, 033001 (2008) [[arXiv:0710.1598 \[gr-qc\]](#)].